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SENSITIVITY ANALYSIS FOR STRUCTURAL DAMAGE DETECTION THROUGH STRAIN ENERGY

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ABSTRACT

This paper proposes a method for structural damage detection through the sensitivity analysis of modal shapes in the calculation of modal strain energy (MSE). For this purpose, sensitivity equations were solved to determine the strain energy based on dynamic data (*i.e.*, modal shapes). An objective function was then presented through the sensitivity-based MSE to detect structural damage. Due to the nonlinearity of sensitivity equations, the objective function of the proposed formulation can be minimized through the shuffled shepherd optimization algorithm (SSOA). The first few modes were employed for damage detection in solving the inverse problem. The proposed formulation was evaluated in a few numerical examples under different conditions. The numerical results indicated that the proposed formulation was efficient and effective in solving the inverse problem of damage detection. The proposed method not only minimized sensitivity to measurement errors but also effectively identified the location and severity of structural damage.

Keywords: Sensitivity analysis; modal strain energy (MSE); damage detection; shuffled shepherd optimization algorithm (SSOA).

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1. INTRODUCTION

Environmental conditions and the passage of time can corrode materials and damage structural elements. Damage to structural elements can alter the behavior and mechanical properties of structures, declining structural performance. The early detection of damage would help reduce costs, *e.g.*, repair and maintenance costs. Therefore, structural health

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monitoring (SHM) and damage detection (damage severity and location identification) by identifying the properties of the structural system are essential in engineering, especially in structural engineering. In other words, damage detection of structures, such as bridges, buildings, and plates, is an important research topic. Researchers often introduce damage as a change in the performance of a structure or that of a structural element. Damage can be evaluated through its location and quantity. Structural damage can occur either gradually or abruptly. Gradual damage includes fatigue and corrosion. It often appears on the surface of elements and then develops under loading and operation, damaging the structural system. Thus, damage can alter structural properties, such as stiffness, strength, and dynamic specifications, in comparison with those of intact structures. As a result, the location and severity of damage to structures appear to be identifiable through structural parameters and dynamic data [1]. Structural damage can be determined by solving the inverse problem through measurements of structural properties and dynamic parameters. Field experiments based on structural behavior under external loads lead to the development of an inverse problem through dynamic data. This approach is efficient and effective in determining minor damage. In some cases, the use of sensitivity analysis in the inverse problem could be an efficient approach to damage detection. Sensitivity analysis is a well-known technique for solving optimization problems where an objective function or design constraints are not the explicit functions of design variables. Sensitivity refers to the rate of change in the structural responses, *e.g.*, displacement or stress, due to a change in the design variables, and the calculation of sensitivity coefficients is known as the sensitivity theory. Sensitivity coefficients are useful in finding the performance sensitivity of structural systems and can serve as guidance in the redesign of a structure [2]. In general, sensitivity analysis can be employed in SHM.

Several techniques have been proposed by developing the inverse problem based on sensitivity analysis for structural damage detection and model updating. Wang and Zhang (1987) used sensitivity analysis and MSE numerically and experimentally to detect damage to a 3D frame. They modeled damage on the element section reduction and used model updating for damage detection. They found that sensitivity to MSE was high, whereas sensitivity to modal shapes was low in damage detection [3]. Gomez and Silva (2008) compared sensitivity analysis-based and optimization-based damage detection techniques. They used modal sensitivity analysis and a genetic algorithm (GA) to detect damage to beams and frames. They showed that both approaches would be efficient in damage localization. However, damage severity identification depended on various variables [4]. Lee (2009) introduced a technique for detecting multiple cracks in a cantilever beam based on the Newton–Raphson method through sensitivity analysis. Their technique assumed the crack location and size as continuous design variables and modeled cracks as rotational springs. The sensitivity matrix was created through the finite difference method (FDM). The results showed good agreement with the real-life model [5]. Kaveh and Zolghadr (2012) studied the damage detection of structures using an inverse problem. They considered the CSS algorithm and proposed a few ideas to improve CSS optimization. The improved and standard CSS algorithms were compared in damage detection. A two-span beam and two 2D frames were evaluated. The results implied that the improved CSS outperformed the standard CSS [6]. Gerist et al. (2012) detected damage to a truss structure and a beam through sensitivity analysis and the continuous genetic algorithm. They evaluated damage

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detection and crack identification by using structural frequencies and static measurements. The framework performance was evaluated by comparing the numerical results [7]. Kaveh and Maniat (2014) used the CSS algorithm and modeled damage detection using vibration data. They exploited natural frequencies and mode shapes to develop the objective function. Penalization was used to reflect noise in the vibration data. The results showed that their method was effective in identifying the location and severity of damage [8]. Kaveh and Zolghadr (2015) analyzed the damage detection of truss structures using the CSS algorithm. They evaluated a 72-member 3D truss and a 10-member 2D truss. Damage detection was formulated as an inverse problem, and the severity of damage to each member was assumed to be a design variable. The objective function of the optimization problem was determined based on the structural frequencies and mode shapes. The results demonstrated that their method was efficient in detecting damage to truss structures [9]. Vo-Duy et al. (2016) proposed a two-step method for detecting damage to laminated composite structures based on MSE. Damaged elements were detected using MSE. An objective function was then defined and minimized based on modal shape errors through the improved differential evolution (IDE) algorithm. The method was applied to a beam and a laminated composite plate. The results implied that the method was efficient in locating damage [10]. Kaveh & Mahdavi (2016) employed the colliding bodies optimization (CBO) and enhanced colliding bodies optimization (ECBO) algorithms to detect damage to truss structures. Analyzing the three numerical examples demonstrated the advantages of ECBO over CBO [11]. Kaveh and Zolghadr (2017) introduced the cyclical parthenogenesis algorithm (CPA) for the damage detection of structures. The damage detection problem was formulated using an index based on modal strain energy (MSE). Furthermore, the objective function was defined using the generalized flexibility matrix (GFM). The CPA was compared to other metaheuristic algorithms and showed superior performance in structural damage detection [12]. Dinh-Cong et al. (2017) introduced a technique based on multi-step optimization through the modified differential evolution (MDE) algorithm. Damage detection would be performed by minimizing the objective function and by using the flexibility matrix of the structure. This technique was evaluated in two numerical examples of laminated composite plates. The results demonstrated that the location and severity of structural damage were effectively identified [13]. Kaveh and Dadras (2018) studied damage detection using noisy and noise-free vibration data and the thermal exchange optimization (TEO) and enhanced thermal exchange optimization (ETEO) algorithms. Damage detection was defined as an inverse problem. The results showed that ETEO outperformed TEO in identifying the damage location and severity [14]. Hamidian et al. (2018) proposed a technique for detecting damage to regular and irregular plates through a combination of wavelet transform and the adoptive neuro fuzzy inference system (ANFIS). This technique would detect damage to irregular plates using only one structural response. Damage was modeled on a reduction in the elasticity modulus, and the structural response was analysed using the 2D wavelet transform. According to the results, the method was found to be efficient in modeling dams [15]. Kaveh et al. (2019) analysed damage detection through a two-phase method based on objective functions in optimization. The objective functions were formulated based on the structural frequency differences and modal shape differences. They also identified the natural frequencies of the structure. The modal shapes were then determined. The second phase would be implemented only when the identified frequencies were consistent. This

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phase substantially reduced the computational time and cost, which were also evaluated. They also used water evaporation optimization (WEO) which proved to be efficient in reducing the computational time and cost [16]. Kaveh et al. (2021) used plasma generation optimization (PGO) to detect damage to skeletal structures. They formulated damage detection as an inverse optimization problem by proposing a combined objective function based on MSE and the flexibility matrix. According to the results, the framework detected damaged elements using only the first few vibration modes even in the presence of noise [17]. Asghari-Motlagh et al. (2021) adopted the continuous genetic algorithm and optimized an onshore platform by considering fatigue damage. They used two optimization scenarios, *i.e.*, CF and WCF, under constraints. According to their findings, the negligence of fatigue could remarkably produce unreliable results in the optimal design of offshore structures, especially platforms [18]. Kaveh et al. (2021) proposed the boundary strategy (BS) for structural damage detection using metaheuristic algorithms. They employed the shuffled shepherd optimization algorithm (SSOA) and evaluated several structural examples to evaluate the BS. The SSOA was also compared to other metaheuristic algorithms in damage detection. The BS decreased the complexity of the search space and accelerated the convergence of the SSOA in identifying the location and severity of damage. The BS technique would be effective for noisy vibration data and large-scale structures [19]. Kaveh et al. (2022) evaluated damage location and severity using the guided water strider algorithm (WSA). They also considered noisy vibration data and evaluated the WSA. A two-stage damage detection method was proposed using MSE and graph-theoretic hierarchical method (GHM). This method exploited a modal strain energy-based index (MBEBI) [20-21]. Damghani and Tavakoli (2023) proposed a method for detecting the location and severity of damage in 2D structures through time-domain responses. They assumed damage to appear in the form of a density reduction and employed the solid isotropic material with penalization (SIMP) method to model the damage. Damage detection was formulated as a topology optimization problem under plane stress conditions. This approach managed to outperform other algorithms [22].

This study proposes a technique for detecting damage location and severity through the MSE and the SSOA. The proposed technique formulates damage detection through sensitivity analysis and the minimized number of vibration modes. Due to the nonlinearity of the sensitivity equations and time-consuming computations, a numerical approach was adopted to obtain the sensitivity matrix. To identify the location and severity of damage, an iterative process in the form of an unconstrained optimization problem was employed. The sensitivity of each element to damage severity was determined, and the sensitivity matrix of the structure was created. An objective function was then developed based on the pre- and post-damage dynamic characteristics to perform optimization. First, a random coefficient in the range of 0~1 was assigned to represent the severity of damage to structural elements. The response vector was then defined based on the proposed formulation for the analytical model of a hypothetical structure. The structure was evaluated under the damage coefficients, and the initially assumed values of the damage were modified through the SSOA. The process continued until the convergence criterion was met. To evaluate the performance of the proposed framework, a truss and two plate structures were analyzed. The results suggested that the proposed framework was efficient and effective.

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2. PROPOSED FORMULATION AND SENSITIVITY MATRIX

2.1 Proposed Formulation for Calculation of Objective Function

The MSE can be defined below, concerning the modal displacement in intact and damaged structures [23].

$$U_i^{h} = \frac{1}{2}\left\{\phi_i^{h}\right\}^T \left[K^{h}\right]\left\{\phi_i^{h}\right\} \tag{1}$$

$$U_i^{h} = \frac{1}{2}\left\{\phi_i^{h}\right\}^T \left[K^{h}\right]\left\{\phi_i^{h}\right\} \tag{2}$$

where U_i^{h} and U_i^{d} represent the strain energy values of mode i in intact and damaged structures, respectively. Moreover, $[K^h]$ and $[K^d]$ denote the stiffness matrices of intact and damaged structures, respectively. Furthermore, $\left\{\phi_i^h\right\}$ and $\left\{\phi_i^d\right\}$ indicate the modal displacements of mode i in intact and damaged structures, respectively. Damage to a structure changes its dynamic parameters and the total strain energy of the system. The change in the total strain energy of the system can be written as below [24].

$$\Delta U = U^{h} - U^{d} \tag{3}$$

where U^{h} and U^{d} represent the total strain energy values of intact and damaged structures, respectively. The stiffness matrix of a damaged structure can be written based on that of the intact structure as below [25].

$$\left[K^{d}\right] = \left[K^{h}\right] - \left[\Delta K\right] \tag{4}$$

where $\left[\Delta K\right]$ denotes the stiffness matrix change. The insertion of Eq. (4) into Eq. (2) yields the MSE of the damaged structure.

$$\begin{aligned} U_i^{d} &= \sum_{el=1}^{n}\left(U_i^{d}\right)_{el} = \frac{1}{2}\sum_{el=1}^{n}\left\{\phi_i^{d}\right\}^T \left[K^{d}\right]_{el}\left\{\phi_i^{d}\right\} = \\ &\frac{1}{2}\sum_{el=1}^{n}\left\{\phi_i^{d}\right\}^T \left[K^{h}\right]_{el}\left\{\phi_i^{d}\right\} - \frac{1}{2}\sum_{el=1}^{n}\left\{\phi_i^{d}\right\}^T \left[\Delta K\right]_{el}\left\{\phi_i^{d}\right\} \end{aligned} \tag{5}$$

where n indicates the number of structural elements. Accordingly, the total strain energy change is defined as the sum of changes in the strain energy of elements. The energy change of an element can be determined by using Eq. (1) and inserting Eq. (5) into Eq. (3).

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$$\begin{aligned}(\Delta U_i)_{el} &= \left((U_i^h)_{el} - (U_i^d)_{el}\right) \\ &= \frac{1}{2}\left(\{\phi_i^h\}^T [K^h]_{el} \{\phi_i^h\} - \{\phi_i^d\}^T [K^h]_{el} \{\phi_i^d\}\right) + \frac{1}{2}\{\phi_i^d\}^T [\Delta K]_{el} \{\phi_i^d\}\end{aligned} \tag{6}$$

Due to insufficient data, the stiffness matrix change $[\Delta K]_{el}$ is unknown. The strain energy change can be determined by using the Taylor series and the approximate values of the initial terms. Therefore, the MSE of element *i* due to structural damage and the stiffness change of element *j* are written as below.

$$\begin{aligned}(\Delta U_i)_{el} &= \sum_j \frac{\partial}{\partial \alpha_j}(U_i^h)_{el}\,\Delta\alpha_j = \\ &\sum_j \left(\frac{\partial \{\phi_i^h\}^T}{\partial \alpha_j}[K^h]_{el}\{\phi_i^h\} + \frac{1}{2}\{\phi_i^h\}^T \frac{\partial [K^h]_{el}}{\partial \alpha_j}\{\phi_i^h\}\right)\Delta\alpha_j\end{aligned} \tag{7}$$

where $\Delta\alpha_j$ denotes the damaged-induced change in the stiffness of element *j*. If damage-induced changes in element *j* do not affect the stiffness matrix of an intact element, the second term in the parenthesis can be assumed zero. Otherwise, it should be included in the calculations. Moreover, $[\Delta K]_{el}$ can be written through the Taylor series as.

$$[\Delta K]_{el} = \sum_j \frac{\partial [K^h]_{el}}{\partial \alpha_j}\Delta\alpha_j \tag{8}$$

Finally, the equality of Eqs. (6) and (7) and the use of Eq. (8) yield the following result:

$$\begin{aligned}&\sum_j \left(\frac{\partial \{\phi_i^h\}^T}{\partial \alpha_j}[K^h]_{el}\{\phi_i^h\} + \frac{1}{2}\{\phi_i^h\}^T \frac{\partial [K^h]_{el}}{\partial \alpha_j}\{\phi_i^h\}\right)\Delta\alpha_j \\ &= \frac{1}{2}\left(\{\phi_i^h\}^T [K^h]_{el} \{\phi_i^h\} - \{\phi_i^d\}^T [K^h]_{el} \{\phi_i^d\}\right) + \frac{1}{2}\{\phi_i^d\}^T \sum_j \frac{\partial [K^h]_{el}}{\partial \alpha_j}\Delta\alpha_j \{\phi_i^d\}\end{aligned} \tag{9}$$

Drawing on matrix compression, Eq. (9) can be abridged as below:

$$[S]\{\alpha\} = \{\Delta R\} \Rightarrow \begin{bmatrix} S_{11} & \cdots & S_{i1} \\ \vdots & \ddots & \vdots \\ S_{1j} & \cdots & S_{ij}\end{bmatrix}\begin{Bmatrix}\alpha_1 \\ \vdots \\ \alpha j\end{Bmatrix} = \begin{Bmatrix}\Delta R_1 \\ \vdots \\ \Delta R_i\end{Bmatrix} \tag{10}$$

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where $\{\alpha\}$ indicates the damage vector, whereas $[S]$ represents the sensitivity matrix, in which the entries are determined through the MSE of each element.

$$S_{ij} = \frac{\partial\{\phi_i^h\}^T}{\partial\alpha_j}\left[K^h\right]_{el}\{\phi_i^h\} + \frac{1}{2}\{\phi_i^h\}^T \frac{\partial\left[K^h\right]_{el}}{\partial\alpha_j}\{\phi_i^h\} - \frac{1}{2}\{\phi_i^d\}^T \frac{\partial\left[K^h\right]_{el}}{\partial\alpha_j}\{\phi_i^d\} \tag{11}$$

where the entries of the residual vector $\{\Delta R\}$ are obtained as:

$$\Delta R_i = \frac{1}{2}\left(\{\phi_i^h\}^T\left[K^h\right]_{el}\{\phi_i^h\} - \{\phi_i^d\}^T\left[K^h\right]_{el}\{\phi_i^d\}\right) \tag{12}$$

The severity of damage was determined by comparing the match between the analytical response and the parameters measured through a set of damage variables. The above system of equations was solved through numerical methods and the objective function of the optimization problem [26].

2.2 Formulating the Sensitivity-Based Optimization Problem

Sensitivity-based damage detection approaches include an optimization problem solved in an iterative process. This study assumed the proposed formulation as two vectors [27].

$$\beta = [S]\{\alpha\} \tag{13}$$

$$\gamma = \Delta R \tag{14}$$

Therefore, the vector form of Eq. (10) is written as:

$$\{\beta\} = \{\gamma\} \tag{15}$$

Considering the vector nature of Eq. (15), the above equality is obtained from the least squares method as below:

$$\varepsilon = \left\|\beta - \gamma\right\|^2 \tag{16}$$

where ε denotes the error in the least squares method, and the optimization process is to minimize the error. Eq. (16) is used as the objective function to find the location and severity of the damage. To solve this function, it would be required to define an iterative unconstrained optimization problem. This study evaluated the unconstrained optimization problem through the SSOA to determine the damage coefficients of each element. As a result, the optimization problem is formulated as below:

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$$\begin{aligned} Find: &\quad \alpha^T = \{\alpha_1, \alpha_2, ..., \alpha_{ne}\} \\ Minimize: &\quad F(\alpha) = \|\beta - \gamma\|^2 \\ Where: &\quad 0 \le \alpha \le 1 \end{aligned} \tag{17}$$

where α^T denotes the damage variable vector, which includes the locations and severity of damage to the structural elements. It is found by solving Eq. (17) through the SSOA. The parameter α ranges from 0 to 1 for each element. A value of 1 represents zero damage, whereas a value of 0 indicates full damage. Damage was modeled on an elasticity modulus reduction in structural elements, and the new elasticity modulus of an element is written as below:

$$E_j^d = \alpha_j E_j^h \quad j = 1, ..., ne \tag{18}$$

where E_j^d and E_j^h refer to the damaged and intact elasticity module of element *j*, respectively. The change in the elasticity modulus of a section would be a better indicator of damage than other parameters, *e.g.*, the moment of inertia and cross-sectional area.

2.3 Shuffled Shepherd Optimization Algorithm (SSOA)

Kaveh and Zaerreza [28-29] proposed the SSOA as a meta-heuristic algorithm inspired by the herding behavior of shepherds in nature. It generates solutions (sheep) and searches the design space through the movement of the sheep based on the shepherd and horse movements. First, the sheep are randomly generated, with each sheep representing a design in the design space. Then, all the sheep are evaluated to determine the competence of each sheep based on the objective function. The sheep are then shuffled into *nh* herds. To this end, *nh* herds are assumed, and the first *nh* sheep are selected and distributed randomly in these herds. Once the first sheep has been allocated to a herd, the allocation process is resumed by selecting *nh* of the remaining sheep. This process continues until all the sheep have been allocated to the *nh* herds. Once the shuffling process has been completed, all the herds have the same number of sheep, and the best and worst sheep in each herd represent the first and last members of the herd, respectively. The movement vector must then be determined in the design space for each design. Based on natural observations, shepherds run the sheep toward the horse. Therefore, the equivalent design of the selected sheep for movement is known as the shepherd $X_{i,j}$. In each herd, the sheep (designs) with better and worse objective function values than the shepherd will be randomly selected. The equivalent designs better and worse than the shepherd are referred to as the horse $X_{i,h}$ and sheep $X_{i,s}$, respectively. To guide the sheep toward the horse, the shepherd changes their position toward the sheep and then toward the horse. Therefore, the movement step of the selected sheep is obtained from the following equation:

$$\begin{aligned} &Stepsize_{i,j} = \alpha \times rand_2 \circ \left(X_{i,s} - X_{i,j}\right) + \beta \times rand_1 \circ \left(X_{i,h} - X_{i,j}\right) \\ &i = 1, 2, ..., nh \quad j = 1, 2, ..., ns / nh \end{aligned} \tag{19}$$

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where *nh* and *ns* denote the number of herds and the number of sheep, respectively. Moreover, $rand_1$ and $rand_2$ represent the vectors whose entries range from 0 to 1. These values are generated randomly between 0 and 1. Here, α and β refer to the control parameters of the SSOA.

$$\alpha = \alpha_0 - \frac{\alpha_0}{\max iteration} \times iteration \tag{20}$$

$$\beta = \beta_{\min} + \frac{\beta_{\max} - \beta_{\min}}{\max iteration} \times iteration \tag{21}$$

where α_0, $\beta_{\max}$, and $\beta_{\min}$ are set by the user. Accordingly, an increase in the number of iterations linearly decreases α to zero, whereas β linearly increases from $\beta_{\min}$ to $\beta_{\max}$ as the number of iterations increases. Once the movement step of all sheep is determined, the new position of each sheep is updated as:

$$X_{i,j}^{new} = X_{i,j}^{old} + Stepsize_{i,j} \quad i = 1,2,...,nh \quad j = 1,2,...,ns/nh \tag{22}$$

Then, $X_{i,j}^{new}$ is obtained using $X_{i,j}^{old}$, with the sheep of higher competence replacing those of lower competence. This process is implemented in all herds. The new herds are then combined, and the sheep are sorted in their competence in descending order, and one iteration of the algorithm is completed. The new iteration begins with the re-shuffling process. The algorithm iterates until the discontinuance criterion, *i.e.*, a predefined number of iterations, is met. Finally, the best sheep is introduced as the optimal design.

3. NUMERICAL EXAMPLES

3.1 Example 1: Square Three-Layered Composite Plate

Fig. 1 demonstrates a square three-layer composite plate analysed to evaluate the performance of the proposed formulation [13]. The plate had a total thickness of 5 *cm*, with each layer having a thickness of 5/3 *cm*. The two materials of the plate had the elasticity module of E_1=40 *MPa* and E_2=1 *MPa* and a shear modulus of G_1=G_2=0.6E_2. The plate was constrained on all sides, and the Poisson's ratio was assumed to be 0.25.

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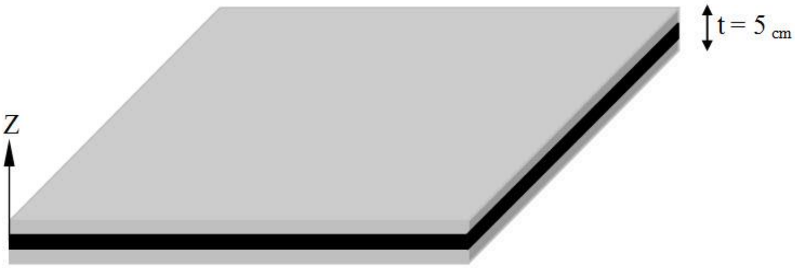

Figure 1. The square 3-layer composite plate

The plate was assumed to have a unit length and a unit width and was meshed into 100 square cells with a size of 10×10 cm^2 (Fig. 2). Two damage scenarios were defined to identify the damage location and severity. Scenario 1 had damaged elements next to each other, whereas Scenario 2 had distributed damaged elements. Fig. 2 illustrates the damage locations in these scenarios. Moreover, only the first three vibration modes were used for damage detection to evaluate the performance of the proposed method. The first, second, and third frequencies of the intact structure were reported 204.97, 408.51 and 642.19 *rad/s*, respectively.

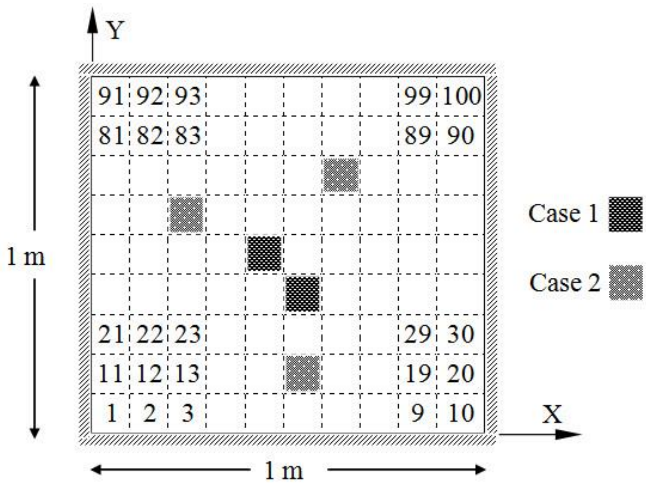

Figure 2. The FE model and damage locations of the three-layer composite plate

3.1.1 Scenario 1

As mentioned earlier, Scenario 1 assumed damaged elements next to each other. Table 1 reports the assumed severity of damage to each element.

Table 1: Damage scenario defined for the 3-layer composite plate (Scenario 1)

Element No.	Damage ratio
36	0.20
45	0.25

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According to Fig. 3, the assumed severity of the damage and the estimated damage severity had good agreement. Therefore, the proposed method effectively identified the damage location; however, it misestimated slight damage to some undamaged elements. For example, elements 75 and 35 were misestimated to be slightly damaged (≈0.05). This error, however, is negligible. Furthermore, a comparison with earlier methodologies [13] demonstrates that the proposed method is efficient and effective.

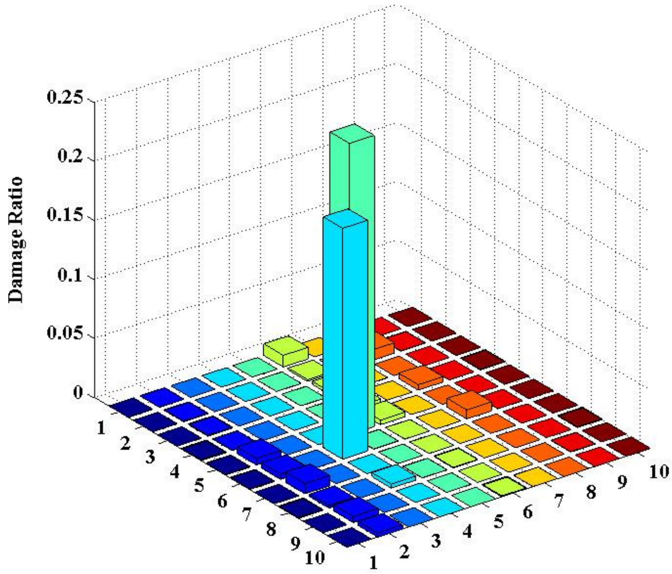

Figure 3. Identified damage elements in scenario 1 for the square 3-layer composite plate

Fig. 4 depicts the convergence of the optimization process. Accordingly, the SSOA was effective in searching the design space and converged on the minimum.

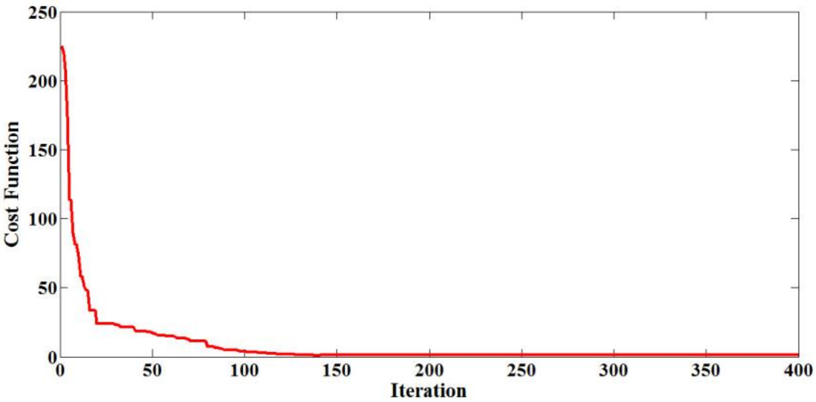

Figure 4. The convergence curves in scenario 1 for the square 3-layer composite plate.

3.1.2 Scenario 2

As mentioned earlier, Scenario 2 assumed distributed damaged elements. Table 2 reports the assumed severity of damage to each element.

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Table 2: Damage scenario defined for the 3-layer composite plate (Scenario 2)

Element No.	Damage ratio
16	0.25
53	0.30
67	0.20

According to Fig. 5, the sensitivity matrix of MSE was constructed concerning the first three modes to detect the damage. The proposed method effectively detected the damage, and the detected damage was in good agreement with the assumed damage with a few elements being misestimated to be slightly damaged (<0.01).

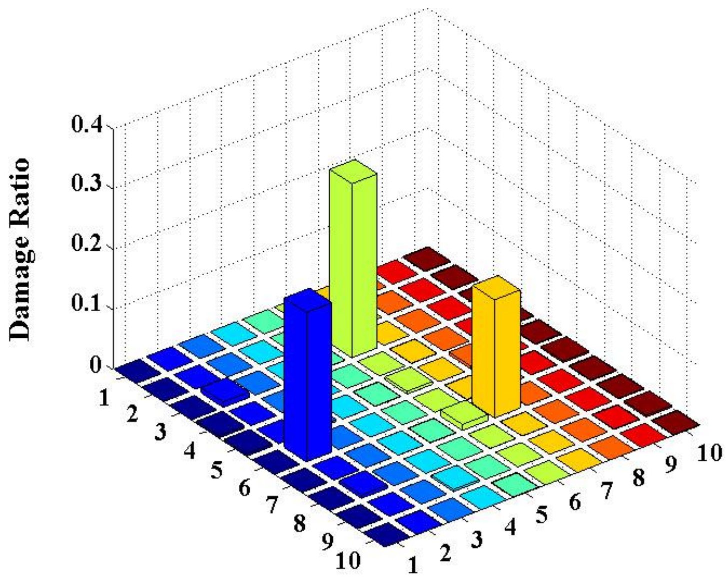

Figure 5. Identified damage elements in scenario 2 for the square 3-layer composite plate

Fig. 6 illustrates the convergence of the optimization process in Scenario 2. The SSOA effectively searched the design space and converged on the optimal solution.

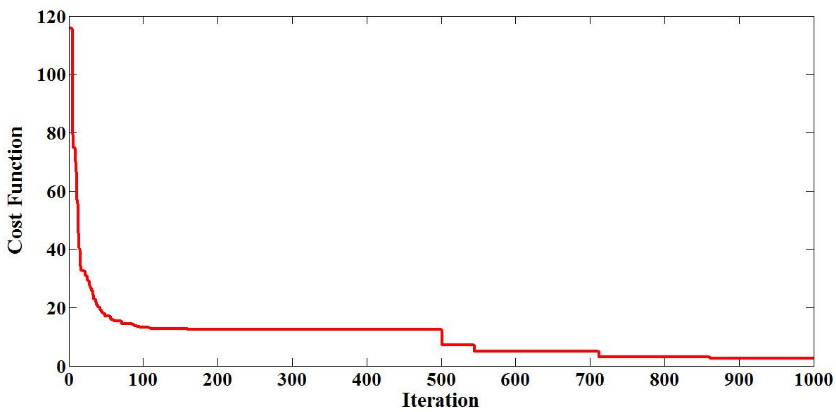

Figure 6. The convergence curves in scenario 2 for the square 3-layer composite plate

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3.2 Example 2: Planar 9-Bar Truss Structure

According to Fig. 7, the proposed formulation was applied to a 9-bar truss [27]. The FE model of the truss had six nodes with nine degrees of freedom (DoFs). The members had an elasticity modulus of E=200 GPa and a density of ρ=7860 kg/m^3. Furthermore, a cross-sectional area of 2.5×10^{-3} m^2 was assumed for the members [27].

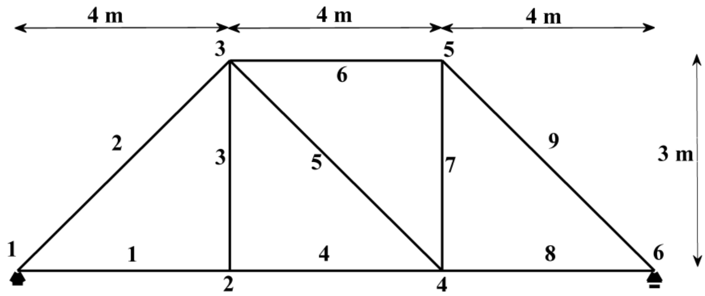

Figure 7. Schematic of the planar 9-bar truss structure

Table 3 shows the damage scenario. It was assumed that only the first vibration mode was available.

Table 3: Damage scenario defined for the planar 9-bar truss structure

Element No.	Damage ratio
3	0.20
8	0.3

Fig. 8 displays the damage detection of the truss structure under the damage scenario. Accordingly, the proposed formulation identified the location and severity of the damage only using the first vibration mode. Based on a comparison with earlier studies [27], the proposed method was efficient and effective in locating the damage.

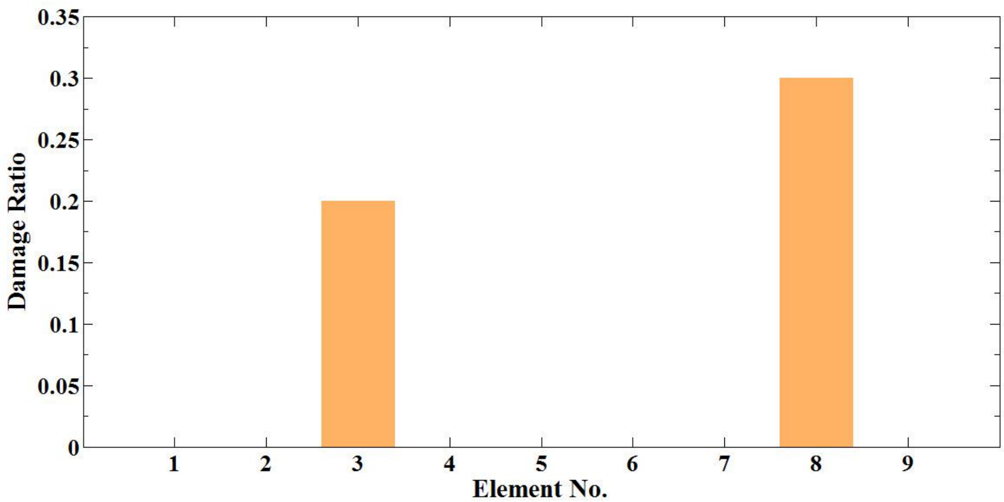

Figure 8. Identified damage elements for the planar 9-bar truss structure

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Fig. 9 indicates the convergence of the optimization process. The SSOA effectively converged on the minimum in the design space.

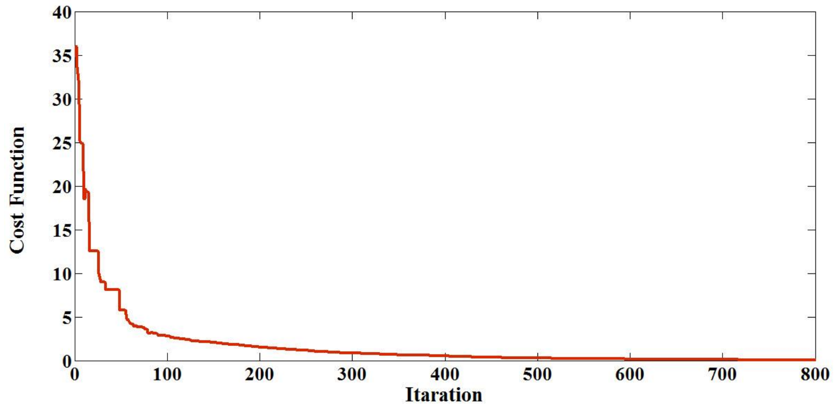

Figure 9. The convergence curves for the planar 9-bar truss structure

3.3 Example 3: Isotropic Rectangular Plate

The proposed formulation was implemented on a 1 *cm* thick rectangular plate with a size of 500×157 cm^2 (Fig. 10). The *E*, Poisson's ratio, and density of the plate were assumed to be 200 *GPa*, 0.3, and 7800 kg/m^3, respectively. The plate was constrained on all four sides. The FE model of the plate was meshed into fifty cells with a size of 50×31.4 cm^2 (Fig. 10).

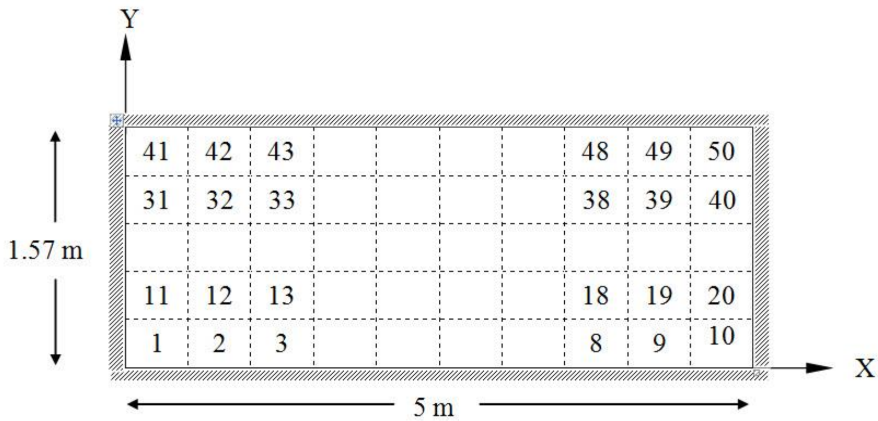

Figure 10. The FE model of the isotropic rectangular plate

Two damage scenarios with slight damage severities were defined to further evaluate the performance of the proposed method. The first three vibration modes were used for damage detection. The first, second, and third frequencies of the intact plate were 25.2, 44.048 and 71.74 *rad/s*, respectively.

3.3.1 Scenario 1

Scenario 1 assumed a slight damage to element 6 (see Table 4). Due to its slight severity, it would be difficult to detect the damage.

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Table 4: Damage scenario defined the isotropic rectangular plate (Scenario 1)

Element No.	Damage ratio
6	0.05

The first, second, and third frequencies were estimated to be 25.19, 44.041 and 71.69 *rad/s*, respectively. Accordingly, the estimated frequencies of all three modes had negligible differences from those of the intact structure. This slight difference resulted from the small severity of the damage to only one element. According to Fig. 11, the damage location was effectively identified. However, the proposed formulation had a small error due to the small severity of the damage and misestimated small damage to other elements. Furthermore, the severity of the damage to element 6 was estimated with a slight error. The use of only the first three modes and the small damage severity led to an insignificant difference between the frequencies of the intact and damaged plates. As a result, damage detection had an error. However, the proposed method detected the damage with an acceptable small error.

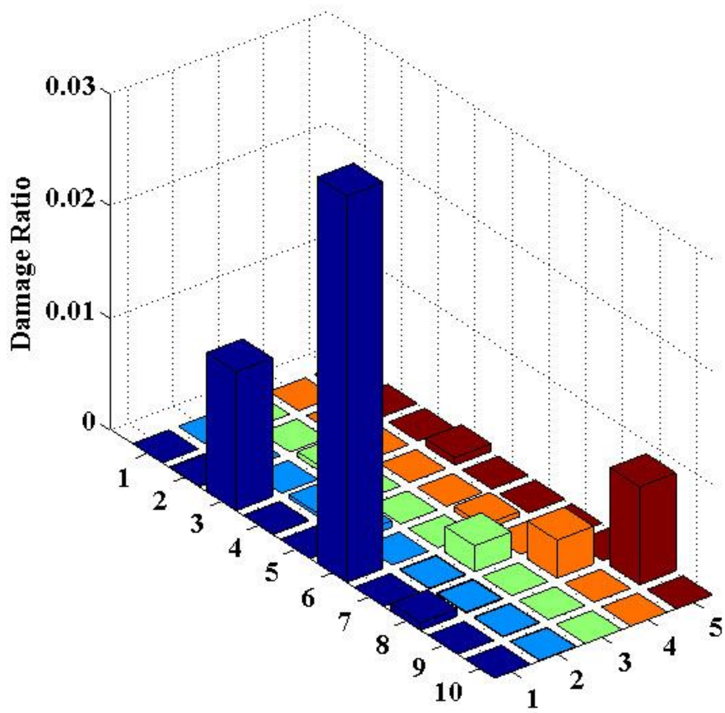

Figure 11. Identified damage elements in scenario 1 for the isotropic rectangular plate

Fig. 12 demonstrates the convergence of the optimization process. According to Fig. 12, the SSOA converged on the minimum at an insignificant error. Failure to reach zero in the convergence diagram was due to the small damage severity and the slight frequency difference between the intact and damaged plates. Therefore, a few undamaged elements were misestimated to be damaged, and the objective function was not reduced to zero.

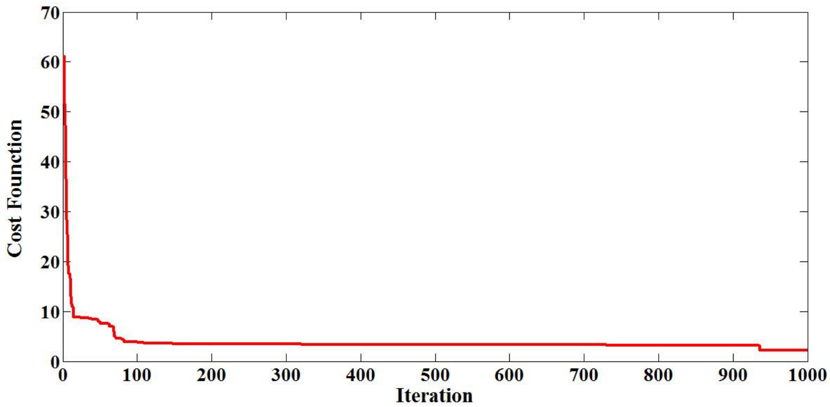

Figure 12. The convergence curves in scenario 1 for the isotropic rectangular plate

3.3.2 Scenario 2

To further evaluate the proposed method, Scenario 2 with small damages was defined for the plate (Table 5). The first, second, and third frequencies were reported 25.13, 43.97, and 71.65 *rad*/*s* for the defined damage, respectively.

Table 5: Damage scenario defined the isotropic rectangular plate (Scenario 2)

Element No.	Damage ratio
6	0.05
29	0.10
41	0.05

Fig. 13 depicts the detection of damage to the plate through the proposed formulation in Scenario 2.

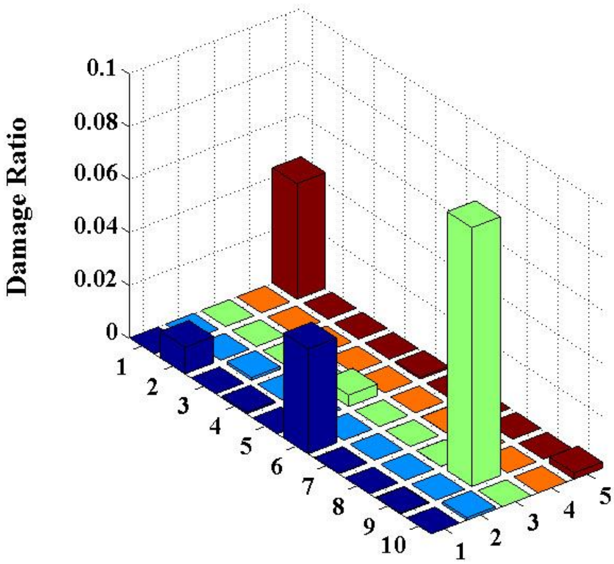

Figure 13. Identified damage elements in scenario 2 for the isotropic rectangular plate

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Accordingly, the damage was detected with good accuracy. Despite the combined damage and the small damage severity, the results had good agreement with the defined scenario. Elements 2 and 25 were estimated to undergo very slight damages, which are negligible (<0.01). In other words, the small error of the proposed method is acceptable concerning the number of modes required for damage detection and the small severity of the damage.

Fig. 14 demonstrates the convergence of the optimization process in Scenario 2. Accordingly, the SSOA effectively searched the design space. It established a good trade-off between global and local searching through its unique search mechanism. However, the objective function did not decrease to zero, as a few undamaged elements were misestimated to be damaged.

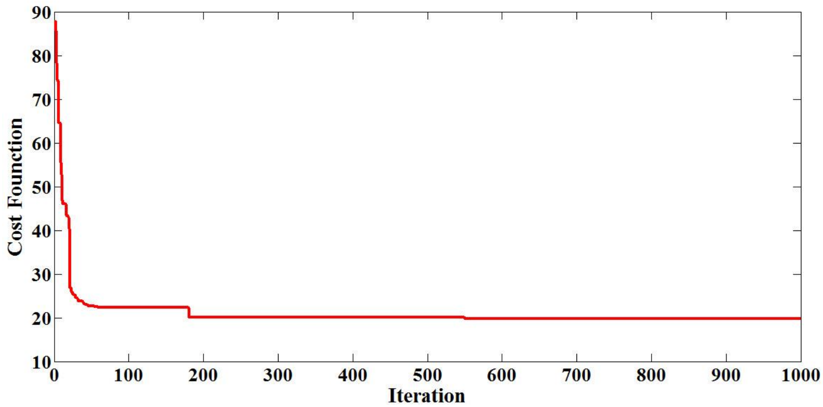

Figure 14. The convergence curves in scenario 2 for the isotropic rectangular plate

4. CONCLUSION

This study aimed to propose a method for identifying the location and severity of damage to structures. Early damage detection helps maintain the functionality of structures. Damage to a structure can alter some dynamic and static properties; therefore, dynamic parameters, *e.g.*, modal characteristics, were used for damage detection. Due to the limited number of dynamic responses, damage detection would be performed through only a few modes. In other words, the main purpose of this study was to develop a damage detection method based on an optimization problem through dynamic data, MSE, and sensitivity analysis. Due to the unavailability of real-life structural data, numerical modelling was conducted. For each example, damage scenarios were defined, analyzing the structure under the assumptions of scenarios. An objective function developed through the sensitivity-based MSE was then employed for damage detection. This objective function was defined concerning damage-induced changes in structural elements, *e.g.*, stiffness and modal shape changes. The SSOA would minimize the objective function, and the location and severity of the damage were identified. The results of a few examples under different damage scenarios indicated that the proposed formulation was efficient and effective. Combined damage and small severities were also included in the damage scenarios. This helped further evaluate the proposed method. According to the findings, the proposed method managed to identify

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damaged elements with a small error and misestimated a few undamaged elements to be slightly damaged. Furthermore, the proposed formulation used the minimum number of structural modes, something which can lead to a shortage of parameters required in constructing the sensitivity matrix and challenge the optimization process. Overall, the SSOA coupled with the proposed formulation effectively searched the design space in the development of the objective function. This process even produced acceptable results under the scenarios with small damage to one element. In other cases, the proposed SSOA-based damage detection framework outperformed earlier methods, despite using fewer dynamic parameters.

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