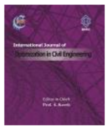
INTERNATIONAL JOURNAL OF OPTIMIZATION IN CIVIL ENGINEERING
Int. J. Optim. Civil Eng., 2024; 14(2):163-172



A GRAPH THEORETICAL APPROACH FOR REGION IDENTIFICATION IN CONTINUUM TOPOLOGY OPTIMIZATION

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ABSTRACT

During the process of continuum topology optimization some pattern discontinuities may arise. It is an important challenge to overcome such irregularities in order to achieve or interpret the true optimal layout. The present work offers an efficient algorithm based on graph theoretical approach regarding density priorities. The developed method can recognize and handle solid continuous regions in a pre-optimized media. An illustrative example shows how the proposed priority guided trees can successfully distinguish the most crucial parts of the continuum during topology optimization.

Keywords: Continuum topology optimization, graph theory, natural associated graph, priority guided tree.

Received: 12 November 2023; Accepted: 17 January 2024

1. INTRODUCTION

Topology optimization of structures have already received attention by several investigators using discrete or continuum approaches [1-5]. The process of continuum topology optimization methods, is commonly based on defining an initial homogenized media covering the whole design domain and then altering its material distribution (subject to total mass constraint) in order to achieve the optimal layout. Consequently, the material content of some elements is forced to approach zero that is, they are eliminated in the final design. As a result, some discontinuous regions may arise in the design domain. They are especially challenging when interpreting the mass-redistributed media in to practical design with a smooth boundary.

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[DOI: 10.22068/ijoce.2024.14.2.580]

A survey of recent continuum optimization methods is given by Bulman *et al*. [1]. Despite various differences in these existing methods, most of them are common in having the potential to produce undesirable layouts such as discontinuous regions or checkerboard patterns. The conditions and reasons for such numerical instabilities to arise are discussed by Diaz and Sigmund [5], and a survey of related procedures is given by Sigmund [6]. They implied the general feature of numerical methods to introduce artificially higher stiffness than actual, especially for lower order finite elements resulting in a periodic pattern of dense and porous material. In that case, the checkerboard patterns appear to be locally stronger, thus being intended in compliance-minimization problems seeking the stiffest structure. Hassani and Tavakkoli [7] used kinematic approach in image processing to extend a noise cleaning technique while Jang et.al. [8] concerned non-conforming elements in order to prevent checkerboard patterns.

Hsu *et.al*. [9] represented a survey of common problems in continuum topology optimization treated by three common category of methods: homogenization-based, material-redistribution and evolutionary optimization techniques. In addition to reviewing the work of Youn and Park [10] in suppressing checker boards, Geudes and Taylor [11] seek high resolution media; while Hsu *et al*. [10] introduced their own interpretation technique based on exploring density contours which are extracted from the result of material density method for topology optimization. They also extended a procedure to identify region discontinuities based on computing the area enclosed by such density contours.

The present work offers a graph theoretical approach to represent computationally efficient algorithms regarding density priorities in order to well recognize and handle stable continuous solid regions in topology of the media. However, other types of priorities can be treated in similar way depending on the optimization technique. A two-dimensional example is treated to illustrate application of the developed method.

2. PRILIMINARY DEFINITIONS

Extensive study of *graph theory* definitions and application in structural mechanics can be found in literature [12-16]. However; some basic terms are reviewed here as follows:

- A *graph* S consists of a non-empty set N(S) of *nodes* (vertices) and a set M(S) of *edges* or *members* together with a *relation of incidence* which associates with each member a pair of nodes called its *ends*.
- Every two ends of a members are considered *adjacent* to each other and *incident* to that member.
- Every two members with a common end are called *incident members.*
- A *subgraph* of a graph consists of subsets of its member set and node set, respectively with the same incidence relationship of the original graph.
- A sequence of alternately non-repeated nodes and members of graph is called a *path* subgraph between its beginning and the ending nodes in such a sequence.
- If the beginning and the end node in such a sequence coincide with each other; the graph it is called a *cycle*.
- A graph is *connected* if and only if there is at least one path between every two nodes of it.

- A *tree* is a connected subgraph with no cycles. A *multi-tree* or *forest* may consist of a number of trees.
- Every connected part of a graph is called its *component.*
- *Skeleton graph* (SKG): a graph associated to a finite element mesh whose nodes and members are in one to one correspondence with the FEM nodes and elements.
- *Natural Associated Graph* (NAG): a graph whose nodes correspond to FEM elements and every two nodes are adjacent if and only if the corresponding finite elements have at least one common node

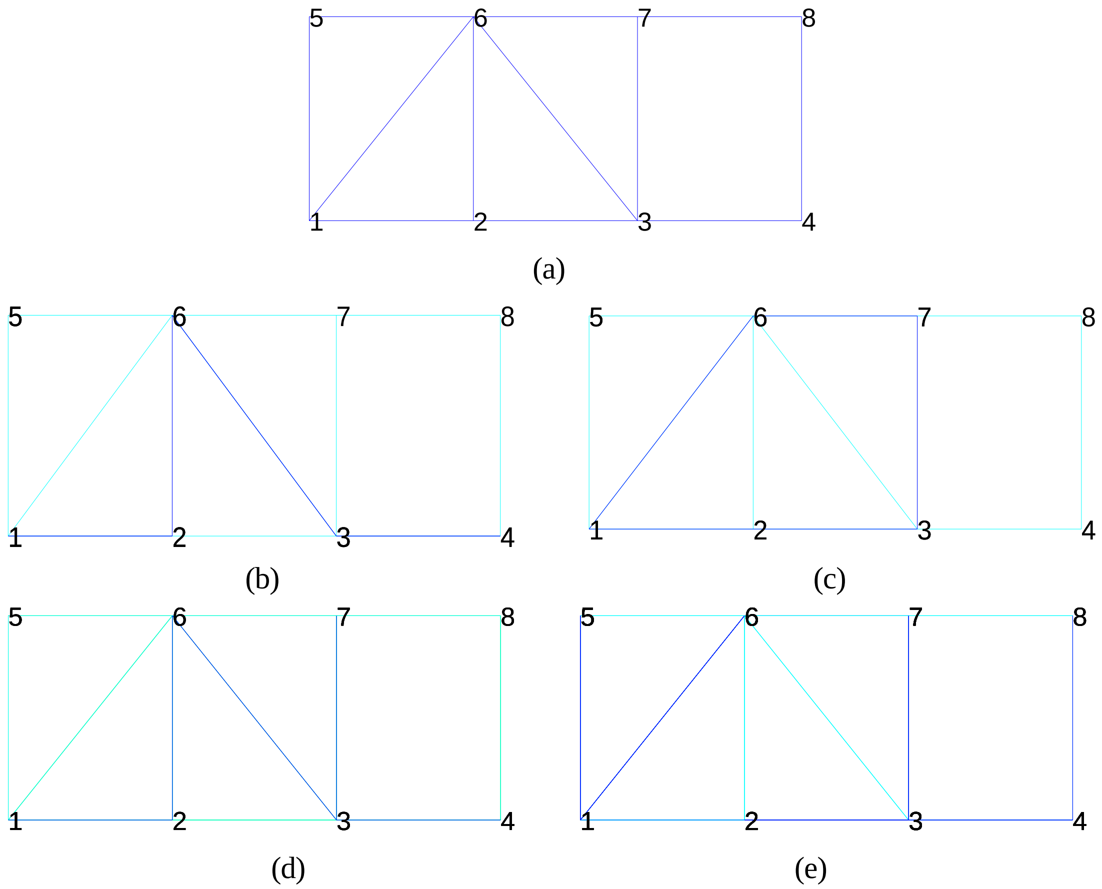


Figure 1. (a) A simple Graph S with 8 nodes and 12 members, (b) a Path subgraph in S, (c) a Cycle subgraph in S, (d) a Tree subgraph in S, (e) a Forest with 2 Trees in S.

Fig.1 demonstrates some popular definitions in a sample graph. A number of extra definitions are developed here as further needed in the solution algorithm.

- *Growing a subgraph*: adding more members (and corresponding nodes) of the graph to the subgraph keeping the same incidence relationship.
- *Priority*: weights associated with the graph nodes in order to guide generation of subgraph(s) on it.

[DOI: 10.22068/ijoce.2024.14.2.580]

- *Priority Guided Tree* (PGT): a (multi)tree subgraph whose nodes are selected as the most priority adjacent nodes as it grows.
- *Stem* : the next node which is to be connected into the PGT.
- *Bud*: a PGT node whose adjacent node (stem) is the next to be connected to the PGT.
- *PGT Root*: the first bud in each of PGT components whose growth is initiated from this bud.
- *Link*: a member connecting stems of two distinct components of a PGT

Every node in a PGT has necessarily its own bud and vice versa. Tracing the path backwards will end to the root node in that component.

3. THE PROPSOED GRAPH-THEORETICAL ALGORITHMS

As locations of the supports and of the exerted loads are usually prone to stress concentration in the continuum, they are expected to earn the most densities during topology optimization. In addition, any final layout design should certainly contain members transferring the loads to the supports. Necessity of existing such intermediate connecting paths emphasizes the presence of support and load point nodes in each possible layout and leads to take them as initial roots of the PGT. Concerning these facts two algorithms are developed as follows to handle the discretized continuum as a *Finite Element Model* (FEM) of the structure. The first algorithm generates the PGT; while the second employs it to handle the region discontinuities by graph theoretical operations.

3.1 The algorithm for generation of the PGT

- Draw the associated graph with the corresponding FEM.
- Assign the corresponding densities as priority numbers to graph nodes.
- Determine the location of supports and load points as initial roots of the PGT.
- Grow the PGT from its roots up to achieve the prescribed mass constraint. At any step of such a growth procedure, only one stem node is connected to its corresponding PGT bud. This stem is selected among all adjacent nodes to those of the current PGT that have not yet entered it, based on possessing maximum priority sum of the generating stem and its bud among all such nodes.

After the PGT growth is completed (the mass constraint is satisfied), identify whether any discontinuous region exists in this media by applying the second algorithm:

3.2 The region identification algorithm using PGT

a. Sort the list of priority guides (PG's) in descending order.
b. Follow this sorted PG history beginning from the maximum PG. For each priority guide, identify the nodes with the same or greater priority outside the PGT as *irregular nodes*. Specify the non-connected nodes among them as *irregular region roots* and connect these roots to the other adjacent nodes in this category; as lower priority limits are considered in the next iterations.
c. Repeat the previous step up to reach the minimum priority guide in the PG history.
d. Identify such selected nodes as discontinuous regions and determine each regional

[DOI: 10.22068/ijoce.2024.14.2.580]

mass and root node.

If the mass of such discontinuous regions are negligible with respect to the total mass constraint, they could be ignored and omitted from the resulted layout to achieve true connected topology. In addition, the PGT could be growth back up to the step with no considerable discontinuous mass, and then the mass constraint or optimization algorithm should be modified to avoid such instability. Employing NAG instead of SKG is also possible as an associated graph in the solution algorithm. In order to avoid checkerboard effects element densities may be initially redistributed. One way is computing corner node's density as average of all its neighbor elements' and then re-computing each element density as averages of values assigned to its corner nodes.

4. THE ILLUSTRATIVE EXAMPLE

As a well-studied benchmark problem, continuum topology optimization of a cantilever beam under a concentrated load on its free end, is addressed (Hsu *et al*.[9]) to interpret the resulting layout. The objective is to minimize the compliance subject to a constraint as 25% mass usage.

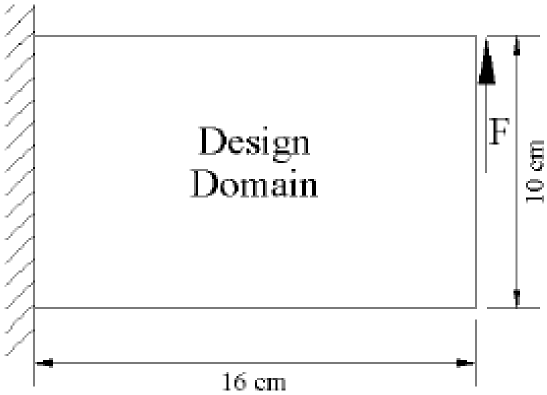


Figure 2. The cantilever beam topology optimization problem under the load (F=500 N) [9].

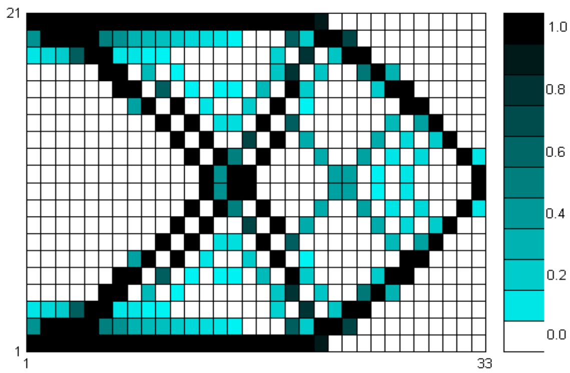
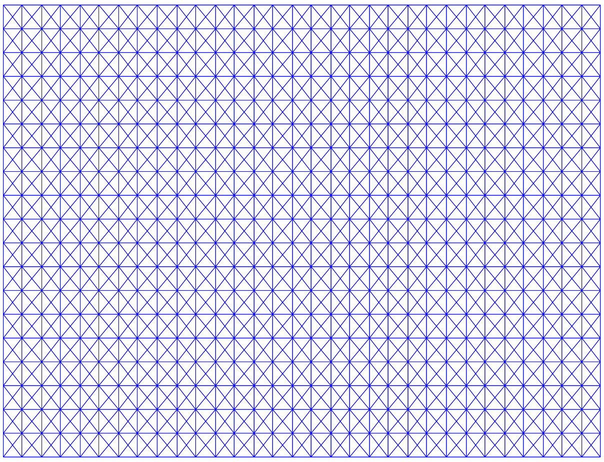


Figure 3. The density distributed media as the topology optimization result to be further interpreted [9]

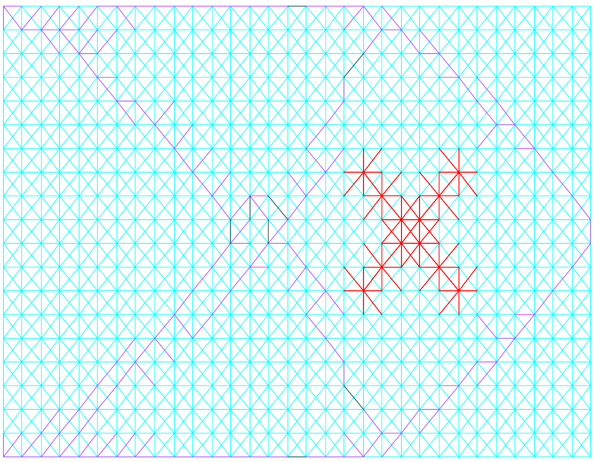
[DOI: 10.22068/ijoce.2024.14.2.580]

The problem dimensions and the design domain are depicted in Fig. 2. The corresponding material density values using a 32x20 finite element grid before pushing them into 0 or 1, are considered for interpretation as depicted in Fig. 3.

Applying the proposed method, the *natural associated graph* for the FEM grid is generated first (Fig. 4(a)). Nodal priorities of the NAG are initiated as the corresponding finite element densities. Then, a PGT is generated on such a natural associated graph from the supporting nodes and the load exertion point as the initial roots (Fig. 4(b)). PGT generation procedure is then continued up to satisfy the fixed mass constraint as 25% of the total design domain volume. Consequently, histories of the priority guide values are obtained and traced in Fig. 5 and Fig. 6.



(a)



(b)

Figure 4. (a) The NAG correspondent to the 32x20 finite element mesh, (b) the resulted Priority Guided Tree (depicted bluish) together with the related links (in black ink) and graph of the discontinuous-region (red).

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Fig.5 reveals that the PGT growth algorithm automatically seeks the most dense elements as its branches grow since the priority guide have remained on its maximum value (unity) for a number of early iterations and then have experience a decreasing trend up to reach a minimum of 0.27 after a few drops of PG. It verifies performance of the proposed algorithm in obtaining true optimal topology and confirms the necessity of selecting support and load points as the initial roots of the PGT. Applying the history of priority variation in the region detection algorithm leads to identify a discontinuous region for the prescribed mass limit in this example. The NAG node with the most priority (the highest element density) at the center of each discontinuous region are notified as the root of graph growing in that region.

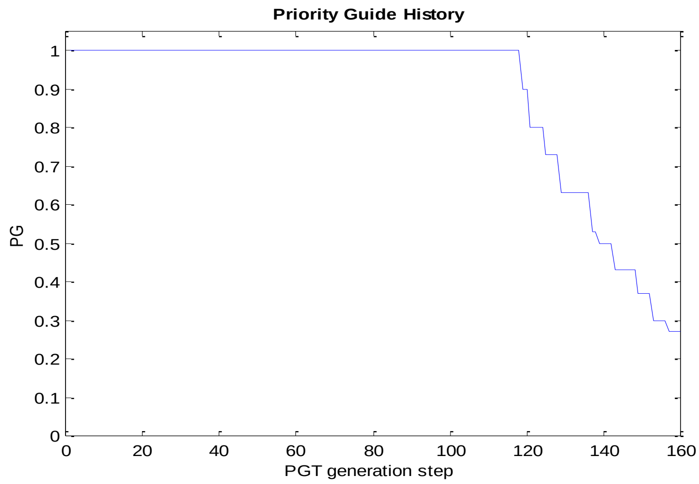

Figure 5. The priority guide trace in the procedure of growing the PGT

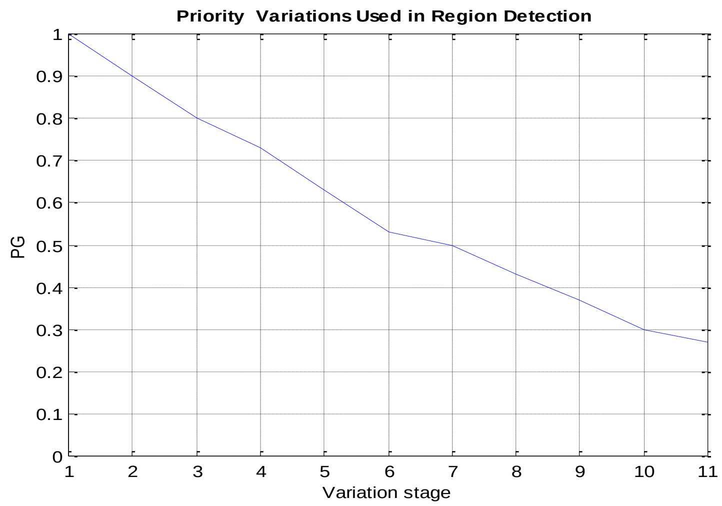

Figure 6. The history of PG variation; decreasing in densities as the PGT grows.

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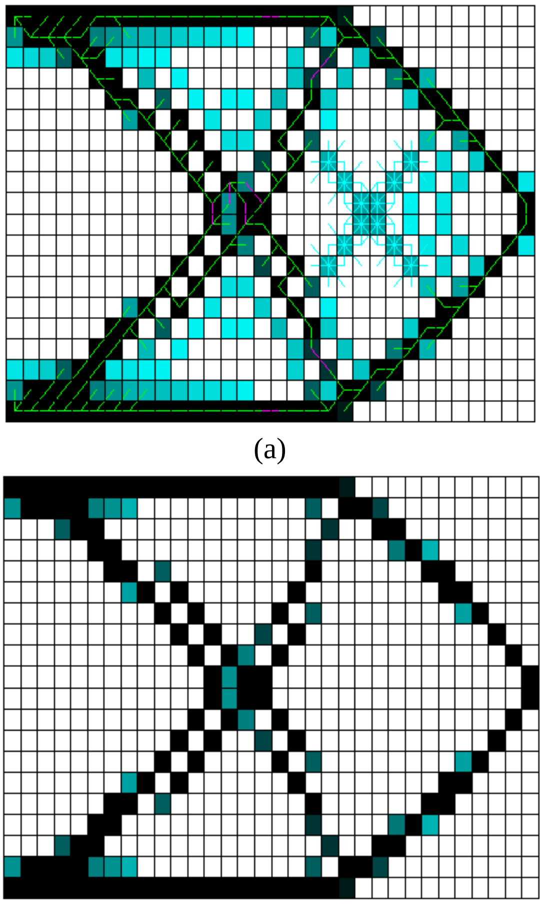

(a)

(b)

Figure 7. (a) The PGT (shown in green lines) mapped on the density distributed media and discontinuous region (in cyan ink), (b) the model after removing the discontionuous region

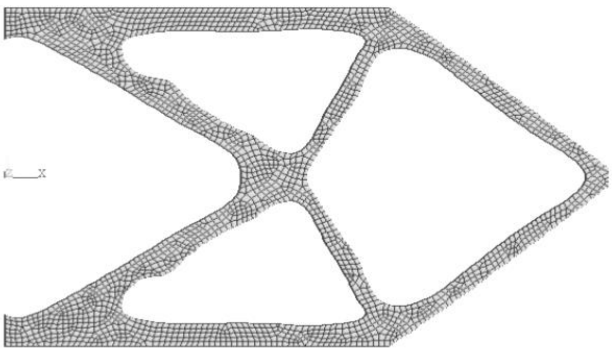

Figure 8. The optimal topology of the cantilever beam [9]

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Fig. 7(a) shows the drawn PGT with link members and discontinuous-region trees on the density distributed background domain for this example. Adequate links are generated automatically between components of the PGT while every component has initialized its growth from one distinct root. In another word, the paths to transfer the imposed load to the support locations, pass through critical links in the PGT.

The result after removing the elements of the irregular regions, is demonstrated in Fig.7(b). Such a result well complies with the final topology solution in literature [9, 17] (see Fig.8) as well as with the regions obtained using density contours by Hsu et.al [9]. It may also be further modified by checkerboard clarifying algorithms and then transferred as initial point to continue shape optimization procedures. On the other hand, the generated graphs by the proposed method, reveal extra topological information such as stem-bud sequence paths and roots.

Note that the number of iterations used to generate PGT is of the order of NAG nodes, i.e., the number of finite elements in the design domain. In addition, since the generation of PGT, is fully combinatorial and independent of the geometry, it can be directly extended to three dimensions.

5. CONCLUSION

The problem of region identification during continuum topology optimization was concerned using graph theory as a powerful tool for handling topology in computer models. To this purpose, new graph theoretical structures were utilized such as priority guided tree and related systems of bud-stem and root nodes. In this case, suitable solution algorithms were developed in order to generate desired patterns in the associated graph model of the FE design domain.

The pre-optimized density distributed media in the well-known example of the cantilever beam from literature was treated to illustrate the application of the proposed method. As a result, the design domain was decomposed into critical connected regions versus discontinuous regions. The fact that the resulted PGT generation history curve did not experience any increase with generation steps emphasized the power of the algorithms in identifying stable solid patterns in the design domain. Target topology design was obtained by removing distinguished irregular regions. It was in agreement with literature works; however, further checkerboard removal is also possible.

An outstanding advantage of the developed procedure is its capability to assess the topology of structural model in a straight forward automatic manner without any need to produce an initial guess or intermediate trials. Furthermore extra topological information such as roots of any priority guided trees as well as irregular region graphs are obtained simultaneously by the proposed method.

Although a two-dimensional problem was treated here, the proposed method can be directly used for three-dimensional problems since it takes the advantage of graph theoretical operations that rely on the topology rather than the geometry of the FE model.

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