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A NEW HYBRID METAHEURISTIC ALGORITHM FOR SIZE OPTIMIZATION OF DISCRETE STRUCTURES

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ABSTRACT

The primary objective of this paper is to propose a novel technique for hybridizing various metaheuristic algorithms to optimize the size of discrete structures. To accomplish this goal, two well-known metaheuristic algorithms, particle swarm optimization (PSO) and enhanced colliding bodies optimization (ECBO) are hybridized to propose a new algorithm called hybrid PSO-ECBO (HPE) algorithm. The performance of the new HPE algorithm is investigated in solving the challenging structural optimization problems of discrete steel trusses and an improvement in results has been achieved. The numerical results demonstrate the superiority of the proposed HPE algorithm over the original versions of PSO, ECBO, and some other algorithms in the literature.

Keywords: Structural optimization, metaheuristic, particle swarm optimization, colliding body optimization, truss structures.

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1. INTRODUCTION

Over the past decade, various optimization algorithms have been extensively used to solve structural optimization problems. Studies indicate that gradient-based algorithms are not suitable for optimal design of structures due to their limitations. [1]. On the other hand, metaheuristics use the random exploration of design space and this makes them more powerful and efficient in solving structural optimization problems, compared to gradient-based ones [2]. It is not necessary for them to have any prior knowledge about the design space or the optimization problem [3]. In recent years, several nature-inspired metaheuristics have been developed, such as simulated annealing [4], genetic algorithms [5], and bacterial foraging [6]. There are other algorithms inspired by social interactions, such as the particle swarm algorithm [7, 8] and ant colonies [9], as well as some inspired by physical laws, such

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as the colliding bodies algorithm [10]. Upon closer inspection, it becomes evident that not all structural optimization algorithms are equally effective in addressing all structural optimization problems. The No Free Lunch Theorem [11] states that no metaheuristic can perfectly solve all types of optimization problems. Proposing and improving metaheuristic algorithms with different computational strategies to tackle different classes of optimization problems is an active area of research [12-16].

The current study proposes a new and efficient approach to tackle discrete structural optimization problems. Instead of introducing a new metaheuristic algorithm, the approach hybridizes two well-known existing algorithms, namely Particle Swarm Optimization (PSO) [7] and Enhanced Colliding Bodies Optimization (ECBO) [17]. The newly proposed metaheuristic algorithm is known as the Hybrid PSO-ECBO (HPE) algorithm. The proposed HPE is a parallel implementation combining both algorithms' benefits without increasing computational cost. To evaluate the effectiveness of the proposed HPE metaheuristic, the steel truss structure optimization problems are considered. The numerical results indicate that HPE outperforms other algorithms in terms of convergence rate and final solutions.

2. PARTICLE SWARM OPTIMIZATION

The PSO algorithm, first proposed by Kennedy and Eberhart [7], is a stochastic algorithm inspired by the social behavior of birds. PSO is an iterative approach to finding optimal solutions for optimization problems, similar to other metaheuristic algorithms. The iteration starts by generating random solutions. These solutions are then updated using a formulation that incorporates current position vectors and velocity vectors of the next stage. The velocity vector of the next stage has three components: the velocity vector of the current stage, an updating vector towards the personal best, and an updating vector towards the global best. It should be noted that these three elements are weighted by some coefficients that are named as the importance factors and must be adjusted to solve a specific optimization problem. The formulation is as follows:

$$X_{i+1}^{j}=X_{i}^{j}+V_{i+1}^{j} \tag{1}$$

where, X and V are the position and velocity vectors, respectively; i refers to iteration number and j is the index of individual solution candidates. In Eq. (1), the velocity of the next stage V_{i+1}^{j} is calculated as follows:

$$V_{i+1}^{j}=\omega V_{i}^{j}+C_1 r_1\left(P_b^{j}-X_i^{j}\right)+C_2 r_2\left(P_g^{j}-X_i^{j}\right) \tag{2}$$

where C_1 and C_2 are self-confidence and global-confidence coefficients respectively; ω is the inertial weight coefficient; P_{b}^{j} and P_{g}^{j} are the best positions experienced by the jth candidate and best position experienced by all candidate solutions up to ith iteration, respectively; r_1 and r_2 are random vectors, chosen from a uniform random distribution between 0 and 1.

3. ENHANCED COLLIDING BODIES OPTIMIZATION

The ECBO metaheuristic algorithm proposed by Kaveh and Ilchi [17] is an improved version of the colliding bodies optimization (CBO) algorithm [10], which uses memory to store some historical best solutions to update the position of the candidate solutions in the design space. The formulation of this algorithm is based on the basic physical concept of the collision of rigid bodies and the change in their position and velocity after the collision. The basic steps of ECBO are as follows [17]:

1. The initial positions of all colliding bodies (CBs) are determined randomly in an *m*-dimensional search space as follows:

$$X_i^0 = X_{\min} + r \times (X_{\max} - X_{\min}) \text{ , } i = 1,2,...,n \tag{3}$$

in which X_i^0 is the initial solution vector of the *i*th CB. Here, $X_{\min}$ and $X_{\max}$ are respectively the lower and upper bounds of design variables; r is a random vector in the interval [0, 1]; n is the number of CBs.

2. The value of mass for each CB is evaluated as follows:

$$m_i = \frac{1}{F(X_i)} \tag{4}$$

where $F(X_i)$ is the objective function value of the *i*th CB.

3. Colliding memory (CM) is utilized to save a number of historically best CB vectors and their related masses. Solution vectors in CM, are added to the population and the same number of current worst CBs are deleted. Finally, CBs are sorted according to their masses.

4. CBs are divided into two equal groups:

(a) Stationary group; $i_S = 1,2,...,\frac{n}{2}$ and (b) Moving group; $i_M = \frac{n}{2}+1, \frac{n}{2}+2,..., n$

5. The velocities of stationary and moving bodies before collision are evaluated as follows:

$$V_{i_S} = 0 \tag{5}$$

$$V_{i_M} = X_{i_S} - X_{i_M} \tag{6}$$

6. The velocities of stationary and moving bodies after collision are evaluated as follows:

$$V'_{i_S} = \left(\frac{(1+\varepsilon)\, m_{i_M}}{m_{i_S} + m_{i_M}} \right) V_{i_M} \tag{7}$$

$$V'_{i_M} = \left(\frac{(m_{i_M} - \varepsilon\, m_{i_S})}{m_{i_S} + m_{i_M}} \right) V_{i_M} \tag{8}$$

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$$\varepsilon = 1 - \frac{t}{t_{\max}} \tag{9}$$

where ε is the coefficient of restitution.

7. The new position of each CB is calculated as follows:

$$X_{i_S}^{\text{new}} = X_{i_S} + \overline{R}_{i_S} \circ V'_{i_S} \tag{10}$$

$$X_{i_M}^{\text{new}} = X_{i_M} + \overline{R}_{i_M} \circ V'_{i_M} \tag{11}$$

where $\overline{R}_{i_S}$ and $\overline{R}_{i_M}$ are random vectors uniformly distributed in the range of [-1,1].

8. A random parameter *pro* is introduced, specifying whether a component of each CB must be changed. For each CB, *pro* is compared with rn_i (i=1,…,n), a random number uniformly distributed within (0, 1). If $rn_i < pro$, one dimension of the ith CB is selected randomly and changed.

9. The optimization process is terminated when a stopping criterion is satisfied.

4. HYBRID PSO-ECBO ALGORITHM

This paper proposes a straightforward and new approach for hybridizing two well-known metaheuristic algorithms: PSO and ECBO. These algorithms simultaneously explore the design space as a parallel implementation strategy. The fundamental steps of the HPE metaheuristic algorithm are as follows:

Step 1. The optimization process starts with generating n random candidate solutions in the design space using Eq. (3).

Step 2. The PSO and ECBO algorithms are simultaneously used to update the position of the particles in the design space. In this case, $2\times n$ updated candidate solutions will be in the design space. Eqs (1) and (2) are used to generate $X_{PSO} = \{X_{PSO}^1\ X_{PSO}^2 \dots X_{PSO}^n\}$ and Eqs (4) to (11) are used to generate $X_{ECBO} = \{X_{ECBO}^1\ X_{ECBO}^2 \dots X_{ECBO}^n\}$.

Step 3. All the updated candidate solutions, $X = [X_{PSO} \quad X_{ECBO}]$, are sorted according to their objective function values. The best n particles (first half) are transferred to the next generation.

$$XS = sort(\{F(X^1)\ \ F(X^2) \dots F(X^{2n})\}) \tag{12}$$

$$X_{new} = \{XS^1\ \ XS^2 \dots XS^n\} \tag{13}$$

Step 4. The optimization process will continue until a termination condition (such as reaching the maximum number of iterations) is met.

Step 5. The current best solution is considered the final solution.

The flowchart of the proposed HPE algorithm is shown in Fig. 1.

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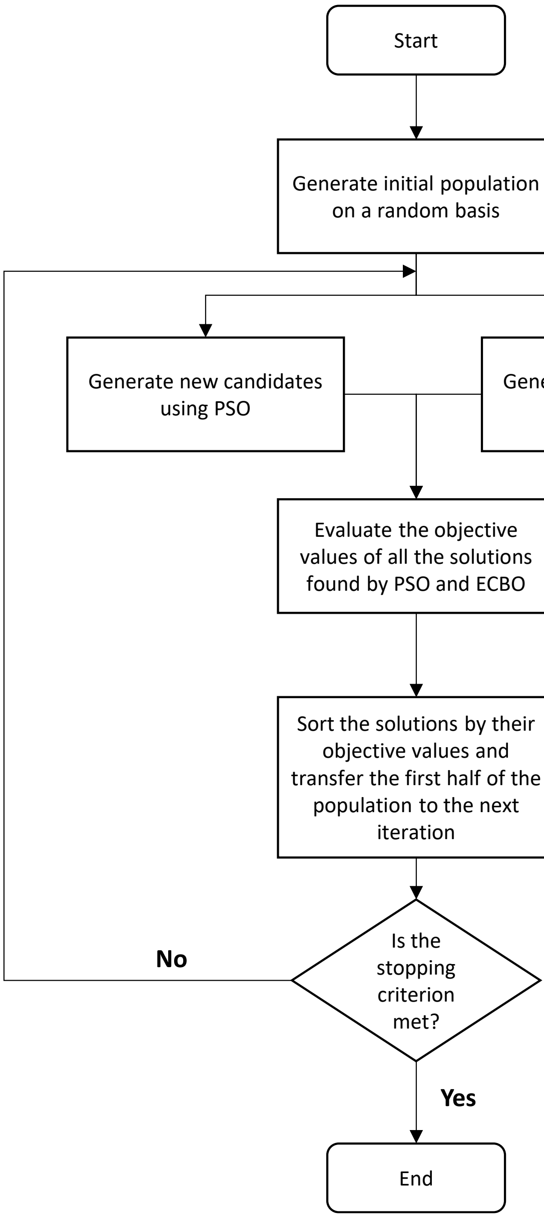

Figure 1. Flowchart of the proposed HPE algorithm

To investigate the efficiency of theproposed HPE metaheuristic algorithm a number of benchmark structural optimization problems are preseted in the next section.

5.ILLUSTRATIVE EXAMPLES

Two types of structural optimization problems are presented: steel truss and RC frame design problems. In the truss design examples, the objective is to minimize the structural weight by selecting cross-sectional areas of elements from a discrete set of available sections. However, in the case of RC frame problems, the main goal is to minimize the construction cost of the frame by selecting cross-sections of beams and columns from predefined standard databases.

5.1 Example 1: Planar 10-bar truss example

Fig. 2 shows the 10-bar truss which is a popular benchmark truss optimization problem. The material density and the modulus of elasticity are 0.1 lb/in.^{3} and 10^4 ksi, respectively. The allowable stress of members and the allowable displacement of all nodes are ±25 ksi and ±2.0 in., respectively. In addition, the magnitude of vertical load *P* is 10^5 lbs. The cross-sectional areas of the structural members are 10 discrete design variables of this design example selected from the following set:

S = {1.62, 1.80, 1.99, 2.13, 2.38, 2.62, 2.63, 2.88, 2.93, 3.09, 3.13, 3.38, 3.47, 3.55, 3.63, 3.84, 3.87, 3.88, 4.18, 4.22, 4.49, 4.59, 4.80, 4.97, 5.12, 5.74,7.22, 7.97, 11.50, 13.50, 13.90, 14.20, 15.50, 16.00, 16.90, 18.80, 19.90, 22.00, 22.90, 26.50, 30.00, 33.50} (in.^{2}).

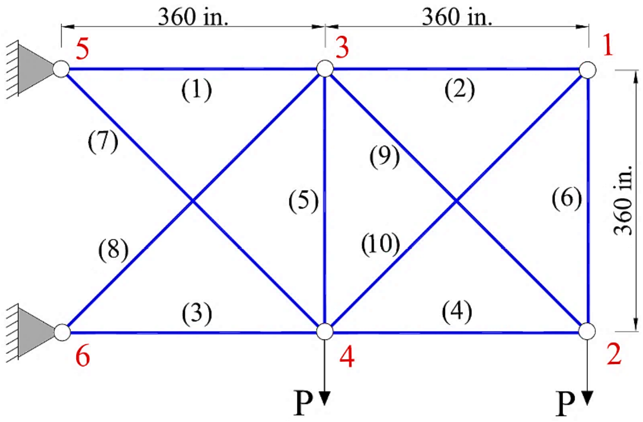

Figure 2. 10-bar truss

In this example, 30 independent optimization runs were performed using different algorithms conducting 4000 structural analyses. Table 1 compares the optimization results obtained using HPE and those reported in the literature. In addition, the best convergence histories of PSO, ECBO and HPE algorithms are compared in Fig. 3 for the 10-bar truss.

Table 1. Optimization results of 10-bar truss

Design Variables (in.^{2})	HHS [18]	SA [19]	BB-BC [20]	GA [21]	HPE
A1	33.50	33.50	33.50	33.50	33.50
A2	1.62	1.62	1.62	1.62	1.62
A3	22.90	22.90	22.90	22.00	22.90
A4	14.20	14.20	14.20	15.50	14.20
A5	1.62	1.62	1.62	1.62	1.62
A6	1.62	1.62	1.62	1.62	1.62

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A7	7.97	7.97	7.97	14.20	7.97
A8	22.90	22.90	22.90	19.90	22.90
A9	22.00	22.00	22.00	19.90	22.00
A10	1.62	1.62	1.62	2.62	1.62
Best (lb)	5490.74	5490.74	5490.74	5613.84	5490.74
Average (lb)	5493.48	N/A	5494.17	N/A	5492.08
Standard deviation (lb)	10.463	N/A	12.420	N/A	6.752
Number of analyses	5000	10500	8694	800	4000

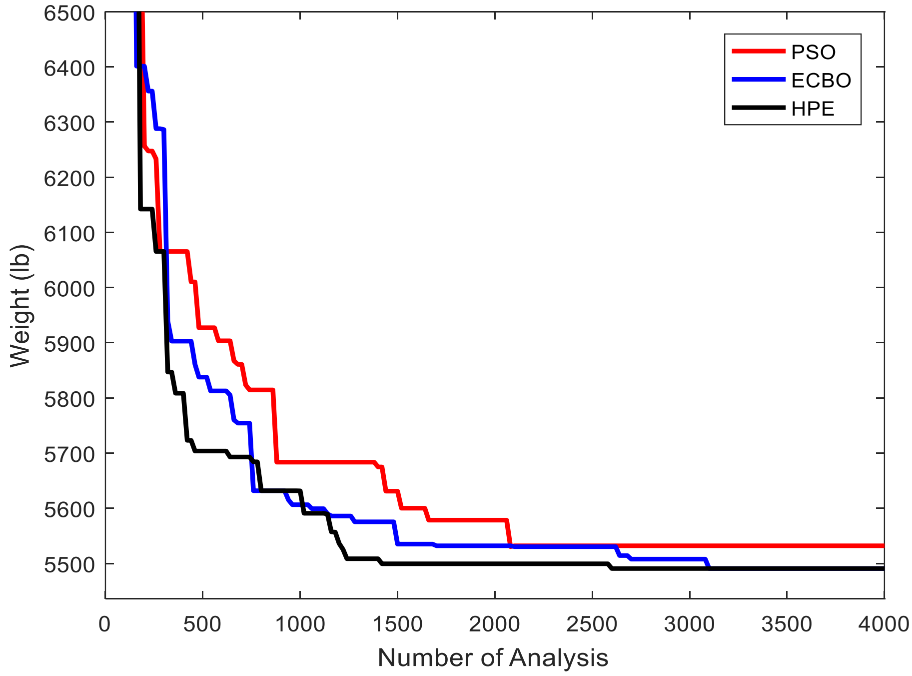

Figure 3. Convergence histories of the best solutions found by PSO, ECBO, and HPE for 10-bar truss

The weights of the best solutions obtained by PSO and ECBO algorithms are 5531.98 and 5490.74, respectively. For ECBO, the values of mean and standard deviation of the weight of the solutions are 5493.084 and 5.752, respectively. On the other hand, the corresponding values for PSO are 5546.48 and 21.873. According to the results, the HPE algorithm outperforms other algorithms.

5.2 72-bar truss example

Fig. 4 shows the 72-bar truss structure considered as the second design example of this paper.

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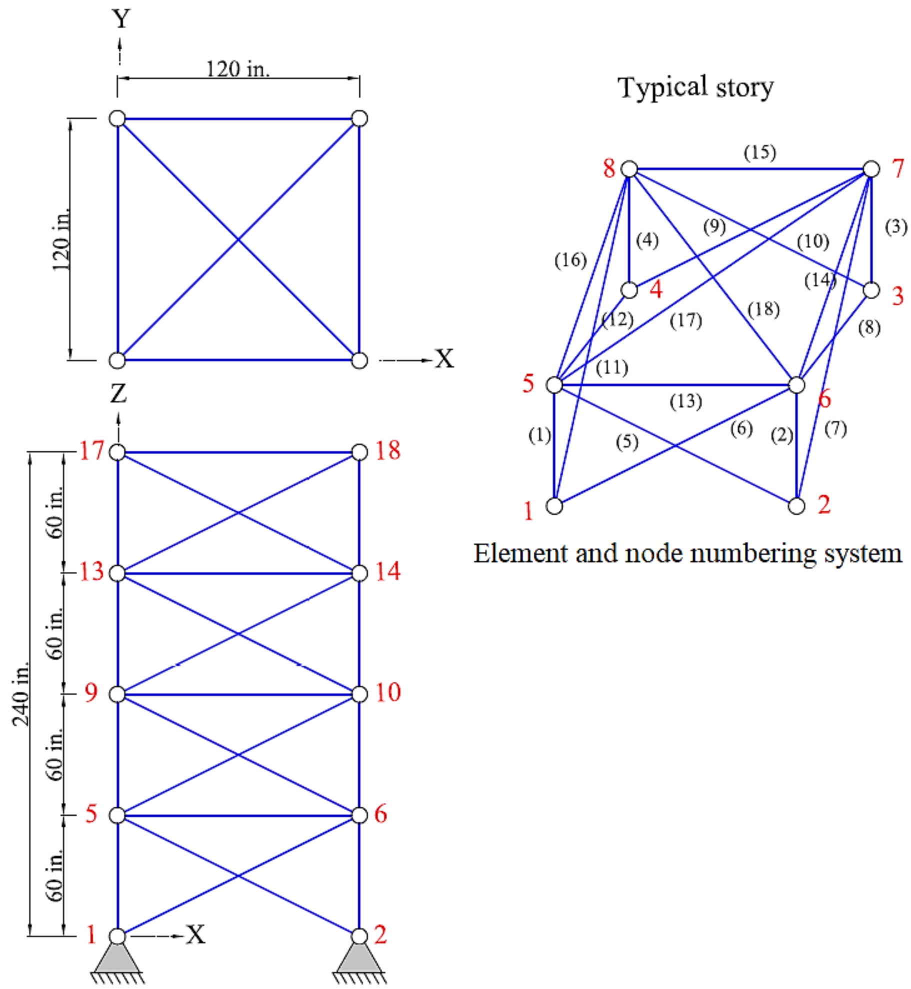

Figure 4. 72-bar truss

The member groups are: (1) A_1–A_4, (2) A_5–A_{12}, (3) A_{13}–A_{16}, (4) A_{17}–A_{18}, (5) A_{19}–A_{22}, (6) A_{23}–A_{30}, (7) A_{31}–A_{34}, (8) A_{35}–A_{36}, (9) A_{37}–A_{40}, (10) A_{41}–A_{48}, (11) A_{49}–A_{52}, (12) A_{53}–A_{54}, (13) A_{55}–A_{58}, (14) A_{59}–A_{66} (15), A_{67}–A_{70}, and (16) A_{71}–A_{72}. The material density and the modulus of elasticity are 0.1 lb/in.^{3} and 10^4 ksi, respectively. There are two independent loading conditions given in Table 2. The nodal displacements and element stresses are limited to ±0.25 in. and ±25 ksi, respectively. The discrete design variables are selected from:

S={0.111, 0.141, 0.196, 0.25, 0.307, 0.391, 0.442, 0.563, 0.602, 0.766, 0.785, 0.994, 1.0, 1.228, 1.266, 1.457, 1.563, 1.62, 1.8, 1.99, 2.13, 2.38, 2.62, 2.63, 2.88, 2.93, 3.09, 3.13, 3.38, 3.47, 3.55, 3.63, 3.84, 3.87, 3.88, 4.18, 4.22, 4.49, 4.59, 4.8, 4.97, 5.12, 5.74, 7.22, 7.97, 8.53, 9.3, 10.85, 11.5, 13.5, 13.9, 14.2, 15.5, 16.0, 16.9, 18.8, 19.9, 22.0, 22.9, 24.5, 26.5, 28.0, 30.0, 33.5} (in.^{2}).

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Table 2. The load cases for the 72-bar spatial truss

Nodes	Load Case 1 (kips)			Load Case 2 (kips)		
	P_x	P_y	P_z	P_x	P_y	P_z
17	5.0	5.0	-5.0	0.0	0.0	-5.0
18	0.0	0.0	0.0	0.0	0.0	-5.0
19	0.0	0.0	0.0	0.0	0.0	-5.0
20	0.0	0.0	0.0	0.0	0.0	-5.0

Table 3. Optimization results of 72-bar truss

Design Variables (in.^{2})	GA [22]	DE [23]	DHPSACO [24]	IDEACO [25]	HPE
1	0.196	2.130	1.800	1.990	1.990
2	0.602	0.442	0.442	0.563	0.563
3	0.307	0.111	0.141	0.111	0.111
4	0.766	0.111	0.111	0.111	0.111
5	0.391	1.457	1.228	1.228	1.228
6	0.391	0.563	0.563	0.442	0.442
7	0.141	0.111	0.111	0.111	0.111
8	0.111	0.111	0.111	0.111	0.111
9	1.800	0.442	0.563	0.563	0.563
10	0.602	0.563	0.563	0.563	0.563
11	0.141	0.111	0.111	0.111	0.111
12	0.307	0.111	0.250	0.111	0.111
13	1.563	0.196	0.196	0.196	0.196
14	0.766	0.563	0.563	0.563	0.563
15	0.141	0.307	0.442	0.391	0.391
16	0.111	0.563	0.563	0.563	0.563
Best (lb)	427.203	391.329	393.380	389.33	389.33
Average (lb)	-	-	-	390.31	389.73
Standard deviation (lb)	-	-	-	1.010	0.824
Number of analyses	-	-	-	10000	8000

In this example, 30 independent optimization runs were performed using different algorithms conducting 8000 structural analyses. The optimization results obtained using HPE and other algorithms are compared in Table 3. The best convergence histories of PSO, ECBO and HPE algorithms are compared in Fig. 5.

For ECBO, the values of the best, mean, and standard deviation of the weights of the optimal solutions are 393.380, 402.524, and 1.082, respectively. On the other hand, the corresponding values for PSO are 434.48, 446.522, and 3.624. The results show the superiority of the HPE algorithm over other algorithms.

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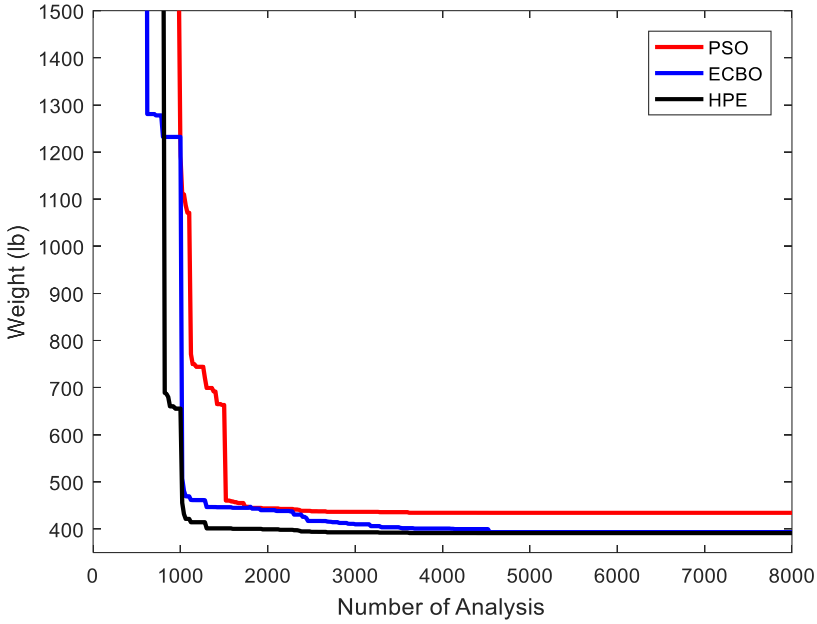

Figure 5. Convergence histories of the best solution found by PSO, ECBO, and HPE for 72-bar truss

5.3 200-bar truss example

A 200-bar truss optimization problem is considered as the third illustrative example of this paper. Fig. 6 shows the geometry and member grouping details of this structure. The material density and the modulus of elasticity are 0.283 lb/in.^{3} and 3×10^4 ksi, respectively. The allowable stress of members is ±10 ksi. There are three loading conditions as follows:

Loading Condition 1: 1 kip load forces act in positive x direction at nodes 1, 6, 15, 20, 29, 34, 43, 48, 57, 62 and 71; *Loading Condition 2*: 10 kips forces act negative y direction at nodes 1, 2, 3, 4, 5, 6, 8, 10, 12, 14, 15, 16, 17, 18, 19, 20, 22, 24, 26, 28, 29, 30, 31, 32, 33, 34, 36, 38, 40, 42, 43, 44, 45, 46, 47, 48, 50, 52, 54, 56, 57, 58, 59, 60, 62, 64, 66, 68, 70, 71, 72, 73, 74, and 75; *Loading Condition 3*: combining loading conditions 1 and 2.

Cross-sectional areas of members are selected from the following set:
S={0.100, 0.347, 0.440, 0.539, 0.954, 1.081, 1.174, 1.133, 1.488, 1.764, 2.142, 2.697, 2.800, 3.131, 3.565, 3.813, 4.805, 5.952, 6.572, 7.192, 8.525, 9.300, 10.850, 13.330, 14.290, 17.170, 19.180, 23.680, 28.080, 33.700} (in.^{2}).

In this example, 30 independent optimization runs were performed using different algorithms conducting 12000 structural analyses. The optimization results obtained using HPE and other algorithms are compared in Table 4. In Fig. 7, the best convergence histories of PSO, ECBO and HPE algorithms are compared.

For ECBO, the values of the best, mean, and standard deviation of the weights of the optimal solutions are 27789.097, 28487.524, and 922.082, respectively. On the other hand, the corresponding values for PSO are 28075.4, 29427.522, and 1232.624. The results show the superiority of the HPE algorithm over other algorithms.

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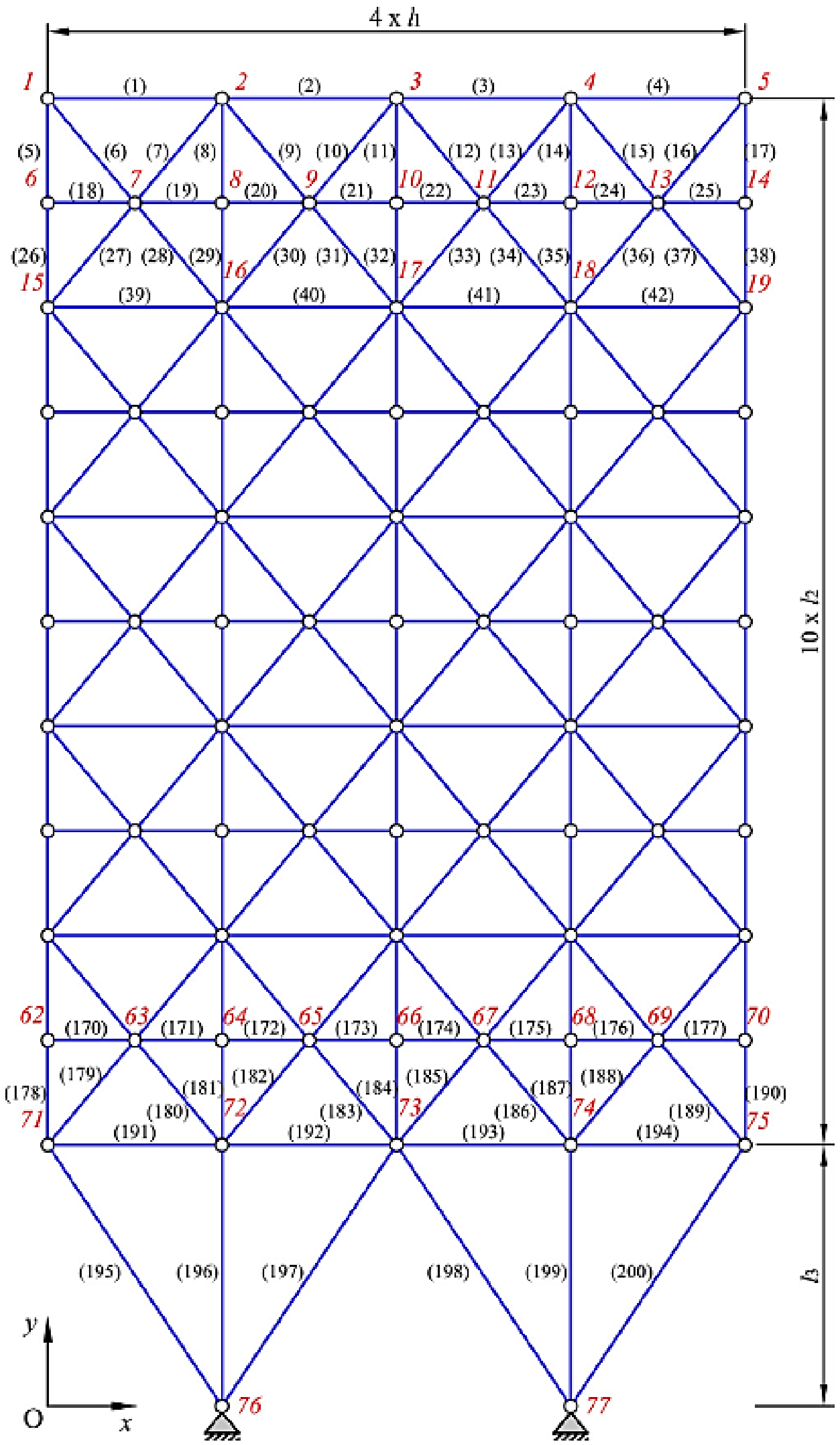

Figure 6. 200-bar truss

Table 5. Optimization results of 200-bar truss

Design Variables (in.^{2})	IMV [26]	mSOS [27]	HPE
A_1	0.1	0.1	0.1
A_2	0.954	0.954	0.954
A_3	0.44	0.44	0.44
A_4	0.1	0.1	0.1
A_5	2.142	2.142	2.142
A_6	0.347	0.347	0.347
A_7	0.1	0.1	0.1
A_8	3.131	3.131	3.131
A_9	0.1	0.1	0.1
A_{10}	4.805	4.805	4.805
A_{11}	0.44	0.44	0.44
A_{12}	0.347	0.44	0.347
A_{13}	5.952	5.952	5.952
A_{14}	0.1	0.1	0.1
A_{15}	6.572	6.572	6.572
A_{16}	0.954	0.954	0.954
A_{17}	0.1	0.347	0.1
A_{18}	8.525	8.525	8.525
A_{19}	0.44	0.1	0.44
A_{20}	9.3	9.3	9.3
A_{21}	0.954	0.954	0.954
A_{22}	0.1	1.174	0.1
A_{23}	13.33	13.33	13.33
A_{24}	0.1	0.44	0.1
A_{25}	13.33	13.33	13.33
A_{26}	0.954	2.142	0.954
A_{27}	5.952	3.813	5.952
A_{28}	10.85	8.525	10.85
A_{29}	14.29	17.17	14.29
Best (lb)	27281.35	27544.191	27281.35
Average (lb)	28771.426	27629.818	27963.32
Standard deviation (lb)	624.026	90.254	603.67
Number of analyses	15000	21675	12000

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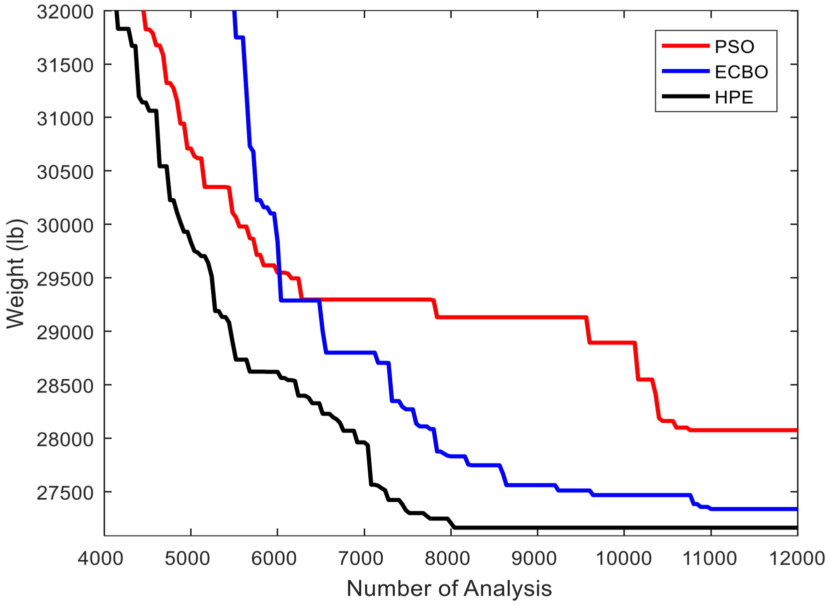

Figure 7. Convergence histories of the best solution found by PSO, ECBO, and HPE for 200-bar truss

6. CONCLUSIONS

This paper proposes a hybrid PSO-ECBO (HPE) algorithm for dealing with discrete structural optimization problems. The HPE strategy combines PSO and ECBO to efficiently explore the design space. It starts by generating random candidate solutions in the design space using PSO and ECBO. The best particles are directly transferred to the next generation after sorting updated candidate solutions according to their objective values. This process continues until a termination condition is satisfied, and the current best solution is considered the final solution.

In order to illustrate the efficiency of the proposed HPE, three well-known discrete benchmark truss optimization problems of 10-, 72-, and 200-bar truss structures are presented. This paper compares the numerical results of HPE in 30 independent runs with those of other metaheuristics including PSO and ECBO. The numerical results indicate that in all the examples, the performance of the proposed HPE is better than the other algorithms.

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