Rotor Design of IPMSM Traction Motor Based on Multi-Objective Optimization using BFGS Method and Train Motion Equations

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Abstract: In this paper a multi-objective optimal design method of Interior Permanent Magnet Synchronous Motor (IPMSM) for traction applications so as to maximize average torque and to minimize torque ripple has been presented. Based on train motion equations and physical properties of train, desired specifications such as steady state speed, rated output power, acceleration time and rated speed of traction motor are related to each other. By considering the same output power, steady state speed, rated voltage, rated current and different acceleration time for a specified train, multi-objective optimal design has been performed by Broyden–Fletcher–Goldfarb–Shanno (BFGS) method and Finite Element Method (FEM) has been chosen as an analysis tool. BFGS method is one of Quasi Newton methods and is counted in classic approaches. Classic optimization methods are appropriate when FEM is applied as an analysis tool and objective function isn't expressed in closed form in terms of optimization variables.

Keywords: Broyden–Fletcher–Goldfarb–Shanno (BFGS) Method, Interior Permanent Magnet Synchronous Motor (IPMSM), Multi-Objective Optimal Design, Quasi Newton Methods.

1 Introduction

Nowadays, since wheel-rail technology in comparison to MAGLEV technology imposes lower cost, is so common over the world. Wheel-rail technology uses rotating motors while MAGLEV technology uses linear motors. Linear electric motors in large scales, lead to more complicated and more expensive structure. IPMSM as a rotating electric motor, due to properties such as high efficiency, high torque density and appropriate flux weakening operation, has been introduced as a competent alternative of induction motor in industry particularly in traction applications. Traction applications require high torque density and low torque ripple [1]. As regards average torque and torque ripple of IPMSM are severely affected by rotor parameters such as permanent magnet dimension and permanent magnet position, multi-objective optimal design of IPMSM in order to approach the mentioned purposes is crucial. To precisely calculate electromagnetic characters such as average torque and torque ripple, finite element analysis is needed. Since in this case there isn't any explicit relation between objective function and optimization variables, classic optimization methods such as BFGS are more appropriate than heuristic methods. Although [2-8] have dealt to optimal design of permanent magnet synchronous motors, it is vital to investigate the desired requirements of application, in particular, requirements of traction applications. Investigation in form of comparison gives a proper view to study the influence of desired requirements on traction motor design. For instance in high speed railway traction system, train physical properties and maximum speed in two cases are considered the same and modification of acceleration time is investigated. So as to perform an appropriate comparison between two cases, particular conditions should be taken into consideration.

In this paper according to train physical specifications, desired system requirements such as acceleration time and maximum speed and based on train motion equations, rated output power and rated speed of traction motor is determined. For two cases of study, only acceleration time is modified. Hence rated speed of two considered cases according to train motion equations are different. By assuming the same rated voltage and rated current for two cases of study, multi-objective optimal design is carried out by BFGS method and finite element analysis is used as an analysis tool.

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2 Problem Definition

The purpose is to optimal design of traction motor corresponding to two case studies tabulated in Table 1. As it is obvious in Table 1, whole parameters for case study1 and case study2 are same except acceleration time. Proposed IPMSM structure considered for investigation has been illustrated by Fig. 1.

Optimal design corresponding to train specifications presented in Table 1 is performed based on train motion equations. By train motion equations and provided train specifications, rated speed corresponding to each case is determined. Subsequently rated torque of traction motor is calculated. Optimization variables include peripheral magnet angle, peripheral magnet width, peripheral magnet thickness, central magnet position relative to rotor center, central magnet thickness and central magnet width which are denoted by x_1, x_2, x_3, x_4, x_5 and x_6, respectively. Optimization variables are illustrated by Fig. 1. According to symmetric property, a pole pitch of proposed IPMSM structure is demonstrated by Fig. 1.

Table 1 Specifications considered for two case studies [9].

Parameter	comment	Case study 1	Case study 2	unit
P_n	Rated propulsion power per a traction motor	250	250	kW
V_{max}	Steady state speed of train	350	350	km/h
GR	Gear ratio	1	1	-
t_f	Train acceleration time	75	47	sec
A_f	Train surface exposed to head wind	10	10	m^2
C_d	Train aerodynamic drag coefficient	1.8	1.8	-
A	Slope angle of train motion path	0	0	deg
M	Train mass per a traction motor	2000	2000	kg
M	Rolling resistance coefficient of steel wheels relative to rail	0.002	0.002	-
D_w	Train wheel diameter	780	780	mm

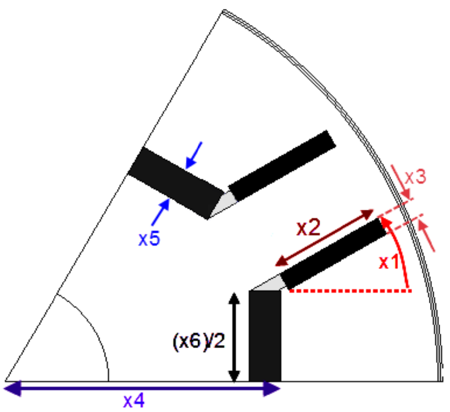

Fig. 1 Proposed rotor structure of IPMSM and optimization variables.

3 Role of Train Operation in Design of Traction System

So as to determine the role of train specifications tabulated in Table 1 in design of traction system, investigation of train motion equation is required. Based on Newton's law, main equation of train could be written as Eq. (1):

$$m\frac{dv}{dt} = F - F_R \tag{1}$$

where m, v, F and F_R stand for train mass per a traction motor, linear speed of train, propulsive force due to a traction motor and resistant force, respectively. Aerodynamic resistant force and rolling resistant force are expressed as Eq. (2) and Eq. (3), respectively [10].

$$F_{aero} = 0.5\rho C_d A_f (V + V_{wind})^2 \tag{2}$$

$$F_{roll} = \mu mg\cos\alpha \tag{3}$$

where ρ, c_d, A_f, V_{wind}, α, g and μ denote air mass density, train aerodynamic coefficient, train surface exposed to head wind, head wind velocity, slope angle of motion path, gravity acceleration and rolling resistance coefficient train wheel relative to rail, respectively.

Resistance force can be calculated by Eq. (4).

$$F_R = F_{aero} + F_{roll} + mg\sin\alpha \tag{4}$$

By substitution of Eqs. (2), (3) and (4) in Eq. (1), acceleration time could be calculated. Train on its determined operation, accelerates from stationary state to steady state speed and continues on this speed. So steady state speed is also maximum speed. It should be noted that Lower than rated speed, maximum torque is constantly exerted on train by traction motors. And over rated speed, exerted torque is proportional to inverse of train speed. In other words, lower than rated speed, Maximum Torque per Ampere control and over rated speed, maximum power control is applied on traction motor. Thus train acceleration time can be expressed as Eq. (5):

$$t_f = \int_0^{V_n} \frac{m}{\frac{P_n}{V_n} - (0.5\rho C_d A_f V^2 + mg(\mu\cos\alpha + \sin\alpha))} dv + \int_{V_n}^{V_{max}} \frac{m}{\frac{P_n}{V} - (0.5\rho C_d A_f V^2 + mg(\mu\cos\alpha + \sin\alpha))} dv\,. \tag{5}$$

where V_n and P_n imply on rated train speed and rated propulsive power per a traction motor respectively. By solving Eq. (5), acceleration time is presented as Eq. (6).

$$t_f = \frac{K_1 K_2^{\,2}}{K_2 - K_3} + K_3 \,\mathrm{Ln}\left|\frac{K_3 - K_2}{K_3 - 1}\right| + K_2 - 1 \tag{6}$$

where K_1, K_2 and K_3 are introduced by Eq. (7), Eq.(8) and Eq. (9) respectively.

$$K_1 = \frac{V_{max}}{(mg^2(\mu\cos\alpha + \sin\alpha))} \tag{7}$$

$$K_2 = \frac{V_n}{V_{max}} \tag{8}$$

$$K_3 = \frac{P_n}{m^2 g^2 V_{max}(\mu\cos\alpha + \sin\alpha)} \tag{9}$$

According to specifications tabulated in Table 1 and presented relations in this part, rated speed of train is calculated then by Eq. (10) rated speed of traction motor in terms of rpm is determined.

$$N = \frac{60\,V_n}{\pi D_w}\,GR \tag{10}$$

Consequently, calculated rated speed of traction motor corresponding to case 1 and case 2 is equal to 2369 rpm and 1200 rpm, respectively.

4 Multi-Objective Optimal Design of Rotor

4.1 Objective Function and Optimization Variables

Since the purpose of optimal design is to maximize average torque and to minimize torque ripple, objective function is proposed as Eq. (11).

$$f = W_1\left(T_{av\,(normalized)} - G_1\right)^2 + W_2\left(T_{ripple\,(normalized)} - G_2\right)^2 \tag{11}$$

where W_1 and W_2 stand for weighting value of average torque and torque ripple that are set to 1 and 2 respectively. G_1 and G_2 imply on desired value of normalized average torque and normalized torque ripple respectively. T_{av} denotes average torque and Tripple denotes torque ripple that is defined as Eq. (12).

$$\mathrm{T_{ripple}} = \frac{T_{max} - \mathrm{T_{min}}}{\mathrm{T_{av}}} \tag{12}$$

Normalized value of average torque and torque ripple should be chosen such a value between 1 up to 10. As such G1 and G2 are considered to 10 and 1 respectively.

Variation interval of introduced variables X_1, X_2 ,X_3, X_4, X_5 and X_6 for two specialized cases are demonstrated in Table 2.

4.2 Multi-objective Optimization by BFGS Method

BFGS optimization method carries out optimization based on second order Taylor expansion of multivariable function f at vicinity of vector X_k and is presented as Eq. (13) [11].

$$\mathrm{m_k(P) = f_k + P^T \nabla f_k + \frac{1}{2} P^T B_k P} \tag{13}$$

where B_k is known as Hessian Matrix of f in K^{th} iteration. By exerting gradient operator to both sides of Eq. (13), minimum point can be stated as Eq. (14):

$$\mathrm{P_k = -B_k^{-1} \nabla f_k = -H_k \nabla f_k} \tag{14}$$

where P_k stands for search direction in k^{th} iteration. in order to calculate P_0, assumption of H_0=I could be acceptable. Eq. (15) represents the updating relation that is exerted on optimizer variables where α_k is determined by line search.

$$X_{k+1}=X_k+\alpha_k P_k \tag{15}$$

By defining S_k and Y_k as Eq. (16) and Eq. (17) respectively, inverse of Hessian matrix in iteration of k+1 is updated by Eq. (18) [11].

$$S_k = \alpha_k P_k = X_{k+1} - X_k \tag{16}$$

$$Y_k = \nabla f_{k+1} - \nabla f_k \tag{17}$$

$$H_{k+1} = H_k + \frac{(S_k^T Y_k + Y_k^T H_k Y_k)(S_k^T S_k)}{(S_k^T Y_k)^2} - \frac{H_k Y_k S_k^T + S_k Y_k^T H_k}{S_k^T Y_k} \tag{18}$$

Table 2 Demonstration of optimization variables for two specialized cases.

Variable	Case study 1		Case study 2		unit
	From	To	From	to	
X_1	25	40	25	40	deg
X_2	19.5	25.5	29	38.5	mm
X_3	2	5.5	3	8.5	mm
X_4	55	65	82	97.5	mm
X_5	5	8.5	7	13	mm
X_6	30	50	44	75	mm

4.3 Optimization Results

By considering the stator specifications for two investigated cases which are tabulated in Table 3 and according to variation interval for optimization variables, multi-objective optimization is performed. Outcome of multi-objective optimization and optimizer values for two studied cases are presented in Table 4.

Table 3 Stator specifications for two investigated cases.

Quantity	Value		unit
	Case study 1	Case study 2	
Stator stack length	504	445	mm
Stator bore diameter	192	288	mm
Stator yoke diameter	283.5	425	mm
Air gap length	2	4	mm
Number of stator slots	36	36	-
Number of poles	6	6	-
Number of turns per phase	48	72	-
Rated phase current (rms)	169.7	169.7	A
Rated phase voltage (rms)	606	606	V
Phase connection	Y	Y	-
Rated frequency	118.45	60	Hz
Type of permanent magnet	Nd-Fe-B	Nd-Fe-B	-
Rated speed	2369	1200	rpm
Maximum speed	2369	2369	rpm
Conductor cross section	26.67	38.59	mm2
Permanent magnet residual flux density	1.23	1.23	T
Relative permeability of magnet	1.09	1.09	-

Table 4 Demonstration of optimization results.

variable	value		unit
	Case study 1	Case study 2	
X_1	30	30	deg
X_2	24.6	36.8	mm
X_3	5	7.5	mm
X_4	59.9	179.7	mm
X_5	6.9	10.4	mm
X_6	42	63	mm
T_{av}	1009	1990	N.m
T_{ripple}	5.82	6.18	Percent

Electromagnetic torque of designed motors corresponding to case study 1 and case study 2 are illustrated by Fig. 2 and Fig. 3 respectively. Since in case study 1, rated speed and maximum speed of traction motor are coincident with each other, for whole speed range, Maximum Torque Per Ampere (MTPA) control is applied on traction motor. In other words, for whole speed range, rated torque is constantly applied on traction motor.

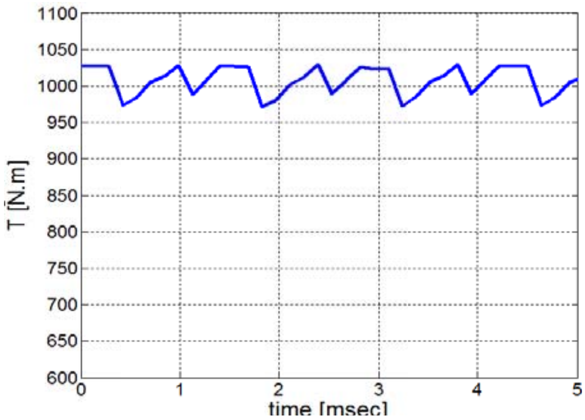

Fig. 2 Electromagnetic torque of designed motors corresponding to case study 1.

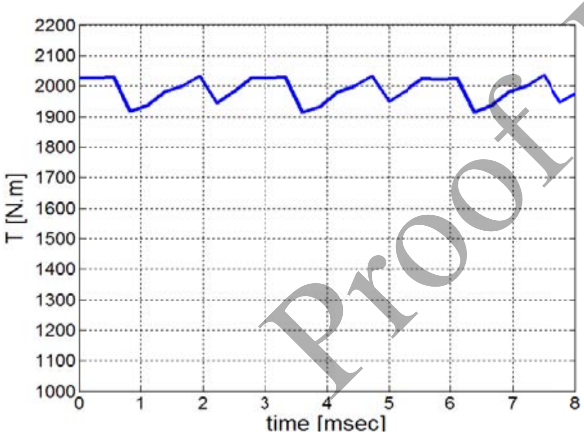

Fig. 3 Electromagnetic torque of designed motors corresponding to case study 2.

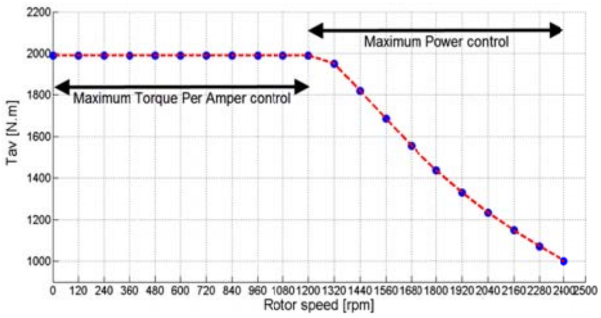

Fig. 4 Torque versus speed in case study 2.

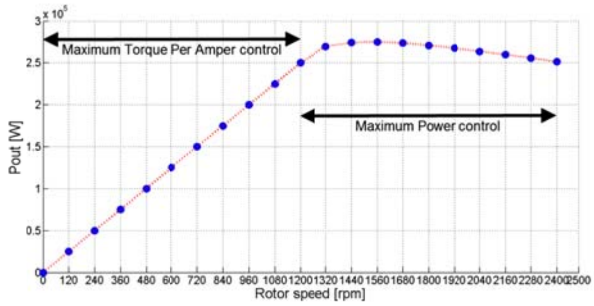

Fig. 5 Output power versus speed in case study 2.

For case study 2, lower than rated speed, Maximum Torque per Ampere control and over rated speed, maximum power control is applied on traction motor. Torque versus speed and output power versus speed for case study 2, have been illustrated by Figs. 4 and 5, respectively.

5 Conclusion

In this paper, multi-objective optimal design of IPMSM for traction application has been presented. Subsequently according to train physical specifications, desired requirements such as acceleration time and steady state speed and based on train motion equations, relation between rated power and rated speed of traction and desired requirements has been represented. For a train with two different acceleration time and by considering same transformers capacity and power electronics convertors, the lower acceleration time, the lower rated speed. By optimizing the average torque, efficiency would be indirectly optimized because optimized average torque is greater than rated average torque, consequently it's possible to reduce the length of motor. Hence, iron loss, copper loss, motor weight and motor volume would be decreased. As regards related output power for traction motors in two cases is same, lower acceleration time leads to greater rated average torque. By determining rating values of traction motor, multi-objective optimal design of rotor is carried out by BFGS method. As regards FEM is selected as an analysis tool, BFGS method is appropriate as a classic optimization method. The proposed multi-objective optimal design method is applicable for whole structures of IPMSM. Also the presented procedure takes into consideration desired requirements of a practical traction system.

References

[1] K. Ch. Kim, J. Lee, H. J. Kim and D. H. Koo, "Multi-objective optimal design for interior permanent magnet synchronous motor", *IEEE Transactions on Magnetics*, Vol. 45, No. 3, pp. 1780-1783, Mar. 2009.

[2] Y. Okamoto, Y. Tominaga, S. Wakao and S. Sato, "Topology optimization of rotor core combined with identification of current phase

angle in IPM motor using multistep genetic algorithm", *IEEE Transactions on Magnetics*, Vol. 50, No. 2, pp. 725-728, Feb. 2014.

[3] F. Parasiliti, M. Villani, S. Lucidi and F. Rinaldi, "Finite-element-based multi-objective design optimization procedure of interior permanent magnet synchronous motors for wide constant-power region operation", *IEEE Transactions on Industrial Electronics,* Vol. 59, No. 6, pp. 2503-2514, Jun. 2012.

[4] Y. Okamoto, Y. Tominaga and S. Sato, "Topological design for 3-D optimization using the combination of multistep genetic algorithm with design space reduction and nonconforming mesh connection", *IEEE Transactions on Magnetics,* Vol. 48, No. 2, pp. 515–518, Feb. 2012.

[5] K. Yamazaki and H. Ishigami, "Rotor-shape optimization of interiorpermanent-magnet motors to reduce harmonic iron losses", *IEEE Transactions on Industrial Electronics,* Vol. 57, No. 1, pp. 61-69, Jan. 2010.

[6] J. Kwack, S. Min and J. P. Hong, "Optimal stator design of interior permanent magnet motor to reduce torque ripple using the level set method", *IEEE Transactions on Magnetics,* Vol. 46, No. 6, pp. 2108–2111, Jun. 2010.

[7] T. Ohnishi and N. Takahashi, "Optimal design of efficient IPM motor using finite element method", *IEEE Transactions on Magnetics*, Vol. 36, No. 5, pp. 3537-3539, Sep. 2000.

[8] D. H. Cho, H. K. Jung and D. J. Sim, "Multi-objective optimal design of interior permanent magnet synchronous motors considering improved core loss formula", *IEEE Transactions on Energy Convesion,* Vol. 14, No. 4, pp. 1347–1352, Dec. 1999.

[9] K. Kondo and M. Kondo, "Design of the parameters of permanent magnet synchronous motors for railway vehicle traction", *The Fifth International Conference on Power Electronics and Drive Systems, PEDS*, Vol. 2, pp. 992–997, Nov. 2003.

[10] K. H. Nam, *AC motor control and electrical vehicle applications*, CRC Press, 2010.

[11] J. Nocdal and S. Jj.Wrijht, *Numerical Optimization*, 2nd Edition, Springer Verlag, 2006.

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