Terminal Sliding Mode Control for Nonlinear Systems with Both Matched and Unmatched Uncertainties

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Abstract: In this paper, we extend the sliding mode idea to a class of unmatched uncertain variable structure systems. This method is achieved with introducing a new terminal sliding variable and the finite time stability of proposed method is proved using a new particular finite time condition in both reaching and sliding phases. In reaching phase a new sliding mode controller is derived to guarantee the finite time stability of sliding surface with considering matched uncertainty. Also in sliding phase, because of introducing a new terminal sliding variable, the finite time stability of state variables with considering unmatched uncertainty has been guarantee. Therefore in proposed algorithm we are able to adjust reaching and sliding times in the presences of both matched and unmatched uncertainty. This algorithm is applied to designing control law for a moving cart system with bounded matched and unmatched uncertainties. Simulation results show the effectiveness and robustness of the proposed algorithm.

Keywords: Nonlinear Systems, Robust Control, Terminal Sliding Mode Control, Unmatched Uncertainty.

1 Introduction

Controlling of uncertain systems is one of the challenges of the modern control theory. Sliding mode control is one of the robust and effective methods to cope with uncertain conditions. Sliding mode control is designed to drive the system states to the so called sliding surface [1-3]. One of the major advantages of sliding mode control is that when the system states are on the sliding surface, the closed loop system behavior is robust to certain parameter variations and disturbances. Sling mode control is well known in the robust control theory for its attractive features such as fast response, good transient property, and the insensitive to the variations of the system parameters and disturbances satisfying the matching condition [4-7].

In the literature, Sliding Mode Control (SMC) has been widely used for a number of applications. In [8] an improved sliding mode control with perturbation estimation featuring a PID-type sliding variable and adaptive gains for the motion tracking control of a micromanipulator system is proposed. In [9], an adaptive sliding mode controller for a class of fractional order chaotic systems with uncertainty and external disturbance is proposed to realize chaos control. This sliding mode controller is shown to guarantee asymptotical stability of the considered fractional order chaotic systems in the presence of uncertainty and external disturbance. Finally in [10, 11] optimal and second order sliding mode control is used for solving the guidance problem.

In conventional SMC algorithm, the most commonly used sliding variable is the linear which is based on linear combination of the system errors by using an appropriate coefficient. Then the sliding mode controller is designed which drive the system to reach and remain on the linear sliding surface in finite reaching time. The gains of the sliding mode controller can be adjusted such that the sliding variable convergent to zero in desired finite time, however, the system states in the sliding mode cannot convergent to zero in finite time. In other words sliding mode control has finite-time convergence to a sliding surface, or is finite-time in reaching phase only. However, for high precision control, the asymptotical stability may not deliver a fast convergence without imposing strong control force [12-17].

Finite time stable systems have not only faster convergence but also better robustness and disturbance rejection properties. It is well known that finite time stabilization of dynamical systems may provide faster disturbance attenuation besides giving faster

convergence to the desired position [18, 19]. Accomplishing finite time error convergence is more desirable in practice. Instead of using a linear sliding variable, Terminal Sliding Mode Control (TSMC) with a nonlinear sliding variable is present. The terminal sliding mode was developed by adding the nonlinear fractional power item into the sliding variable in sliding phase to offer some superior properties, such as finite time convergence of state variables, faster and better tracking precision. Also nonlinear sliding variable in TSMC can improve the transient performance statically [12-17], [20]. The continuous time terminal sliding mode control and discretization of continuous time TSM are analyzed in [16]. In [21] a terminal sliding mode observer is proposed for a class of nonlinear systems to achieve finite time convergence for all error states. Compared to standard sliding mode observers which only enable finite time convergence of the output error, the observer in this paper makes use of fractional powers to reduce other errors to zero in finite time. A 2 degree of freedom robotic manipulator is used to demonstrate the effectiveness of this observer. In [14] an adaptive terminal sliding mode control for DC–DC buck converters has been presented and the purpose of the [20] is to introduce the adaptive TSM controller subject to input nonlinearity for complete synchronization and anti-synchronization between two chaotic gyros, under the existence of system uncertainties and external disturbances. In [15] derivative and integral terminal sliding mode control is presents for a class of MIMO nonlinear systems with first to higher order dynamics. In [17] a fast terminal dynamics is proposed and used in the design of the sliding mode control for SISO nonlinear systems and in [22] fast TSM control is used for a helicopter. In [23] second order fast terminal sliding mode control scheme is proposed to suppress the chaotic motion of a micro mechanical resonator with system uncertainty and external disturbance. In [24] a fractional terminal sliding mode control is introduced for a class of dynamical systems subject to uncertainties. A fractional order sliding variable is proposed and the corresponding control law is designed based on the Lyapunov stability theory to guarantee the sliding condition.

Sliding mode control satisfies the matched uncertainty condition only. During the sliding mode, if the uncertainties of the system satisfy the so-called matching condition, the system behavior has an invariant property which is independent of matched uncertainties. If the matching condition is not satisfied or the system suffers from unmatched uncertainties, then the system behavior in the sliding mode is not only governed by the switching surface but also determined by the unmatched uncertainties. In this case, the system stability may not be assured [25, 26]. The idea of conventional sliding mode from matched uncertain variable structure systems to unmatched uncertain variable structure systems is extend in [25, 26] that are

finite time in reaching phase. And in [27], the nonsingular terminal sliding mode control is developed for MIMO linear systems with unmatched uncertainties.

In this paper, we extend the idea of terminal sliding mode from matched uncertain nonlinear systems to unmatched uncertain nonlinear systems. A new nonlinear switching variable is introduced that guarantee finite time convergence of state variables in sliding mode in the presence of unmatched uncertainty. Then a sliding mode controller is designed to guarantee the finite time reaching to nonlinear sliding surface in the presence of matched uncertainty. This is to say that, first the sliding mode controller can curb the state trajectory to the nonlinear sliding surface in finite reaching phase time and then because of introducing the particular nonlinear sliding surface, the state variables converge to zero in finite sliding phase time.

The paper is organized as follows. In section 2 the sliding mode control theory is introduced and then in section 3 new terminal sliding mode control is proposed. In section 4 proposed algorithm is used to designing control law for an example and numerical simulation results are shown. Conclusions are reported in section 5.

2 Conventional and Terminal SMC

Consider a nonlinear system:

$$X^{(n)} = f(X) + u + w \quad , |w| \le \alpha \quad (1)$$

where $f(X)$ is a known nonlinear part, w is a bounded uncertainty, X is system state vector and u is the control input. Then sliding variable is:

$$S = \left(\frac{d}{dt} + \lambda\right)^{n-1} \tilde{X}, \quad \tilde{X} = X - X_d \quad (2)$$

where λ is a strictly positive constant and X_d is desired state. Assume that u in Eq. (1) must be designed such that the following system has the desired properties and system state reaches to desired state. The tracking problem for $X = X_d$ is equivalent to making $S = 0$. Conventional SMC makes S equal to zero in finite time and then maintain the condition $S = 0$ for all future time. Typical SMC consists of a reaching mode, during which the sliding variable moves to the sliding surface, and a sliding mode, during which the sliding variable is confined to the sliding surface and the S has no variation from sliding surface in system without uncertainty. In conventional SMC the control input is designed as follow:

$$u = u_{eq} + u_{reach}, \quad u_{reach} = -kSign(S) \quad (3)$$

where u_{eq} is the equivalent control determined to cancel the known terms on first derivation of S in system without uncertainty. If there is no uncertainty in the system, the equivalent control $u = u_{eq}$ will maintain the system in the sliding surface. If uncertainties exist, a

sufficient condition to guarantee the finite time attractiveness of $S=0$ for $S\neq0$, is to ensure:

$$\dot{V} = S\dot{S} \leq -\eta|S| \qquad (4)$$

where η is a positive constant, which implies that [1-3]:

$$t_{reach} \leq \frac{|S(0)|}{\eta} \qquad (5)$$

Now consider a nonlinear system with relative degree 2 and matched uncertainty as follow:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x_1,x_2) + g(x_1,x_2)u + w \end{aligned} \quad ,|w| \leq \alpha \qquad (6)$$

For stabilization with conventional SMC, the linear sliding variable is introduced as:

$$S = x_2 + \lambda x_1 \qquad (7)$$

Now assume that the sliding mode controller is able to reaching to sliding surface in finite reaching time. Therefore we yields:

$$\begin{aligned} S = 0 &\Rightarrow x_2 = -\lambda x_1 \Rightarrow \\ \dot{x}_1 &= x_2 \Rightarrow \dot{x}_1 = -\lambda x_1 \end{aligned} \qquad (8)$$

It's mean that the state variable x_1 is exponentially stable. For finite time stabilizing the state variables, terminal sliding mode control is designed with introducing nonlinear sliding variable as follow [12]:

$$S = x_2 + \beta x_1^{p/q} \qquad (9)$$

where $\beta > 0$ is a design constant, both p and q are positive odd integers and satisfy the following condition:

$$1 < p/q < 2 \qquad (10)$$

Another equivalent form of the TSM manifold can be expressed as follows [28]:

$$S = x_2 + \beta|x_1|^{\alpha} sign(x_1) \qquad (11)$$

where $0 < \alpha < 1$.

When the sliding variable (9) reaches to sliding surface $S = 0$, the motion of the system can be described by the following nonlinear differential equation:

$$x_2 = -\beta x_1^{p/q} \Rightarrow \dot{x}_1 = -\beta x_1^{p/q} \qquad (12)$$

where t_s (sliding time) can be calculated as follows:

$$t_s = \frac{p}{\beta(p-q)} x_1^{1-q/p}(t_r) \qquad (13)$$

and t_r is the time when S reaches zero from an initial condition.

The nonlinear sliding variables Eqs. (9) and (11) guarantees the finite time stabilization in sliding phase but are proper to use for systems with matched uncertainty. In next section we introduce a new terminal sliding variable for stabilizing sliding phase in systems with unmatched uncertainty.

3 Terminal SMC for Systems with both Matched and Unmatched Uncertainty

Consider a nonlinear system with relative degree 2 with both matched and unmatched uncertainty as follow:

$$\begin{aligned} \dot{x}_1 &= x_2 + w_1 \quad ,|w_1| \leq \alpha_1 \\ \dot{x}_2 &= f(x_1,x_2) + w_2 + u \quad ,|w_2| \leq \alpha_2 \end{aligned} \qquad (14)$$

where x_1 and x_2 are state variables, $f(x_1,x_2)$ is known nonlinear part, w_1 is bounded unmatched uncertainty, w_2 is bounded matched uncertainty and u is control input. Let us introduce a sliding variable:

$$S = x_2 + \lambda_1 x_1 |x_1|^{\gamma_1 - 1} \qquad (15)$$

Assume that the system is on sliding surface and we are in sliding mode. Therefore yields:

$$S = 0 \Rightarrow x_2 = -\lambda_1 x_1 |x_1|^{\gamma_1 - 1} \qquad (16)$$

Substituting Eq. (16) in Eq. (14) yields:

$$\dot{x}_1 = w_1 - \lambda_1 x_1 |x_1|^{\gamma_1 - 1} \qquad (17)$$

By introducing Lyapunov candidate function:

$$V_1 = \frac{1}{2} x_1^2 \qquad (18)$$

We are able to prove the finite time stability of state variable x_1 using theorem 1.

Theorem 1: A sufficient condition to guarantee the finite time attractiveness of state variable x_1 for $x_1 \neq 0$, is to ensure:

$$\dot{V}_1 \leq -\eta_1 |x_1|^{1+\gamma_1} \qquad (19)$$

where $0 < \gamma_1 < 1$ and η_1 are strictly positive constants, which implies that:

$$t_s \leq \frac{|x_1(t_r)|^{1-\gamma_1}}{\eta_1(1-\gamma_1)} \qquad (20)$$

and t_s is the time required to zeroing x_1.

Proof: by Substituting Eq. (17) in Eq. (19) yields:

$$x_1(w_1 - \lambda_1 x_1 |x_1|^{\gamma_1-1}) \leq -\eta_1 |x_1|^{1+\gamma_1} \Rightarrow$$

$$w_1 \frac{x_1}{|x_1|^{1+\gamma_1}} - \lambda_1 \frac{(x_1)^2 |x_1|^{\gamma_1-1}}{|x_1|^{1+\gamma_1}} + \eta_1 \leq 0 \Rightarrow$$

$$w_1 \frac{x_1}{|x_1|} |x_1|^{-\gamma_1} - \lambda_1 + \eta_1 \leq 0 \Rightarrow \qquad (21)$$

$$\lambda_1 \geq w_1 \frac{x_1}{|x_1|} |x_1|^{-\gamma_1} + \eta_1$$

By choosing λ_1 in Eq. (21) to be large enough, we can now guarantee that Eq. (19) is verified. So that, letting from equation Eq. (21) yields:

$$\lambda_1 = \alpha_1 |x_1|^{-\gamma_1} + \eta_1 \qquad (22)$$

where α_1 denotes the bound of w_1. Also Eq. (20) is proved as follow:

$$\dot{V}_1 = x_1 \dot{x}_1 =$$

$$x_1 \dot{x}_1 \frac{|x_1|}{|x_1|} = |x_1| \frac{d|x_1|}{dt} \le -\eta_1 |x_1|^{1+\gamma_1} \Rightarrow$$

$$\int_{|x_1(t_r)|}^{0} |x_1|^{-\gamma_1} \, d\,|x_1| \le \int_0^{t_s} -\eta_1 dt \Rightarrow \qquad (23)$$

$$\frac{|x_1|^{1-\gamma_1}}{1-\gamma_1} \Bigg|_{|x_1(t_r)|}^{0} \le -\eta t \Big|_0^{t_s} \Rightarrow$$

$$-\frac{|x_1(t_r)|^{1-\gamma_1}}{1-\gamma_1} \le -\eta t_s \Rightarrow t_s \le \frac{|x_1(t_r)|^{1-\gamma_1}}{\eta(1-\gamma_1)}$$

Therefore the state variable x_1 in finite time Eq. (20) reaches to zero with condition Eq. (22) and theorem 1 is proved.

Now we require a controller for stabilizing reaching mode and reaching sliding variable Eq. (15) to sliding surface $S=0$ in finite time t_r. Let a Lyapunov function candidate be:

$$V_2 = \frac{1}{2} S^2 \qquad (24)$$

and a sufficient condition to guarantee the finite time attractiveness of sliding variable S for $S \ne 0$, is to ensure:

$$\dot{V}_2 \le -\eta_2 |S|^{1+\gamma_2} \qquad (25)$$

Let us take the control input as:

$$u = u_{eq} + u_{reaching} \qquad (26)$$

where u_{eq} is the equivalent control determined to cancel the known terms on first derivation of sliding variable in system without uncertainty as follows:

$$\dot{S} = \dot{x}_2 + \dot{f}_1 + \lambda_1 \frac{\dot{x}_1 |x_1|^{1-\gamma_1} - (1-\gamma_1)|x_1|^{-\gamma_1} \dot{x}_1 \frac{x_1}{|x_1|} x_1}{\left(|x_1|^{1-\gamma_1}\right)^2}$$

$$= f_2 + w_2 + u + \dot{f}_1 + \lambda_1 \frac{\gamma_1 \dot{x}_1}{|x_1|^{1-\gamma_1}} = 0 \Rightarrow \qquad (27)$$

$$u_{eq} = -f_2 - \dot{f}_1 - \lambda_1 \frac{\gamma_1 \dot{x}_1}{|x_1|^{1-\gamma_1}}$$

If there is no uncertainty in system Eq. (14), equivalent control $u = u_{eq}$ will maintain the system in the sliding surface. Now, let us consider the case where uncertainties exist. The reaching control is selected as follow:

$$u_{reching} = -\lambda_2 \frac{S}{|S|^{1-\gamma_2}} \qquad (28)$$

Note that in reaching control Eq. (28) the continues function $S/|S|^{1-\gamma_2}$ has smooth properties than discontinues $Sign(.)$ function that is used in conventional sliding mode [1-3]. Therefore control signal with this reaching part is smooth and low chattering compared with conventional sliding mode control.

A sufficient condition to guarantee the finite time attractiveness of sliding surface $S=0$ for $S \ne 0$, is to ensure Eq. (25) which implies reaching time as:

$$t_r \le \frac{|S(0)|^{1-\gamma_2}}{\eta_2(1-\gamma_2)} \qquad (29)$$

In order to satisfy sliding condition Eq. (25) despite matched uncertainty, Substituting Eqs. (26)-(28) in Eq. (25) yield:

$$\dot{V}_2 = S\dot{S} =$$

$$S\left(f_2 + w_2 + u_{eq} + u_{reaching} + \dot{f}_1 + \lambda_1 \frac{\gamma_1 \dot{x}_1}{|x_1|^{1-\gamma_1}}\right) =$$

$$S\left(w_2 - \lambda_2 \frac{S}{|S|^{1-\gamma_2}}\right) \le -\eta_2 |S|^{1+\gamma_2} \Rightarrow \qquad (30)$$

$$w_2 \frac{S}{|S|^{1+\gamma_2}} - \lambda_2 \frac{S^2}{|S|^{1-\gamma_2}|S|^{1+\gamma_2}} + \eta_2 \le 0 \Rightarrow$$

$$\lambda_2 \ge w_2 \frac{S}{|S|} |S|^{-\gamma_2} + \eta_2$$

By choosing λ_2 in Eq. (30) to be large enough, we can now guarantee that Eq. (25) is verified. So that, letting:

$$\lambda_2 = \alpha_2 |S|^{-\gamma_2} + \eta_2 \qquad (31)$$

where α_2 denotes the bound of w_2. By substituting Eqs. (27), (28) and (31) into Eq. (26), control input is given as:

$$u = -f_2 - \dot{f}_1 - \lambda_1 \frac{\gamma_1 \dot{x}_1}{|x_1|^{1-\gamma_1}} - (\alpha_2 |S|^{-\gamma_2} + \eta_2)\frac{S}{|S|^{1-\gamma_2}} \qquad (32)$$

With this controller it is certain that the trajectory of system converges to the manifold $S = 0$ at finite time and it will be confined to that manifold for all future time. Note that the sliding surface also is designed such that state variable x_1 reaches to zero in finite time Eq. (20) after zeroing sliding variable in finite time Eq. (29). Therefore system Eq. (14) with terminal sliding mode controller Eq. (32) is finite time in reaching and sliding modes.

4 Simulation Example

In this section, the proposed control strategy is applied to a cart moving on a plane (Fig. 1). The structure of the model is the same as in [29], and is represented by:
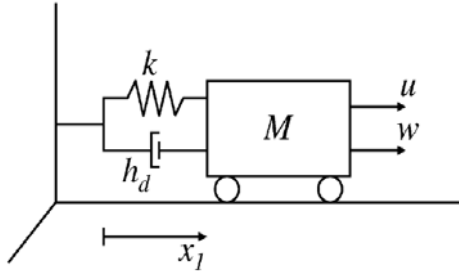
Fig. 1 Graphical representation of the simulation example.

$$\dot{x}_1 = x_2 + w_1 \tag{33}$$

$$\dot{x}_2 = \frac{1}{M}(-k_0 e^{-x_1} x_1 - h_d x_2 + w_2 + u) \tag{34}$$

where x_1 is the displacement of the cart with respect to the equilibrium position, and x_2 is its velocity. M is the mass of the cart, $k = k_0 \exp(-x_1)$ is the stiffness of the spring, and h_d is the damping factor. The control variable is the force u applied to the cart, while w_2 is the load force given by the wind. The presence of another disturbance term w_1 is assumed. The considered system is a control-affine system with matched (w_2) and unmatched (w_1) disturbances [29].

For designing control law, first we introduce a sliding variable as:

$$S = x_2 + \lambda_1 x_1 |x_1|^{\gamma_1 - 1} \tag{35}$$

Finite time reaching displacement of cart in a desired value provides the motivation for this sliding variable. By a suitable design of controller u, if we are able to achieve $S = 0$, thus guaranteeing finite time stabilizing displacement of cart.

First we assume that controller u has been designed and sliding variable is on sliding surface $S = 0$ and we have:

$$x_2 = -\lambda_1 x_1 |x_1|^{\gamma_1 - 1} \tag{36}$$

Therefore the bound of λ_1 for finite time stabilization displacement of cart by satisfying finite time condition Eq. (19) is given as Eq. (22) that guarantee stabilization displacement of cart in the presence of unmatched uncertainty with finite time Eq. (20).

Equivalent control determined to cancel the known terms on the first derivation of sliding variable as follow:

$$\dot{S} = \frac{1}{M}(-k_0 e^{-x_1} x_1 - h_d x_2 + w_2 + u) + \lambda_1 \frac{\gamma_1 \dot{x}_1}{|x_1|^{1-\gamma_1}}$$

$$\Rightarrow u_{eq} = k_0 e^{-x_1} x_1 + h_d x_2 - M \lambda_1 \frac{\gamma_1 \dot{x}_1}{|x_1|^{1-\gamma_1}} \tag{37}$$

If there is no matched uncertainty in the system, then equivalent control will maintain the system in the sliding surface. Now, let us consider the case where uncertainties exist. A sufficient condition to guarantee the finite time attractiveness of sliding surface $S=0$ for

$S \neq 0$, is to ensure Eq. (25) which implies reaching time as Eq. (29). The reaching control is selected as Eq. (28). Then in order to satisfy sliding condition Eq. (25) despite matched uncertainty, λ_2 is achieved as Eq. (31) and control input is given as:

$$u = k_0 e^{-x_1} x_1 + h_d x_2 - M(\alpha_1 |x_1|^{-\gamma_1} + \eta_1)\frac{\gamma_1 \dot{x}_1}{|x_1|^{1-\gamma_1}} -$$

$$M\left(\alpha_2 \left|x_2 + (\alpha_1 |x_1|^{-\gamma_1} + \eta_1)\frac{x_1}{|x_1|^{1-\gamma_1}}\right|^{-\gamma_2} + \eta_2\right) \times$$

$$\frac{x_2 + (\alpha_1 |x_1|^{-\gamma_1} + \eta_1)\frac{x_1}{|x_1|^{1-\gamma_1}}}{\left|x_2 + (\alpha_1 |x_1|^{-\gamma_1} + \eta_1)\frac{x_1}{|x_1|^{1-\gamma_1}}\right|^{1-\gamma_2}} \tag{38}$$

With this controller first the trajectory of system converges to the manifold $S=0$ at finite reaching time and then state variable x_1 reaches to zero in finite sliding time. Therefore system Eqs. (33)-(34) with terminal sliding mode controller Eq. (38) is finite time in reaching and sliding modes.

Now, the proposed terminal sliding mode controller Eq. (38) is applied to a cart moving on a plane (Fig. 1). The structure of the model is the same as in [29], and is represented by Eq. (33) and Eq. (34). The values of the parameters are $M=1$ kg, $k_0=0.33$ N/m, $h_d=1.1$ Ns/m.

### 4.1 Case I: Stabilization of Proposed Algorithm

In this section we address to stabilization with proposed algorithm. The uncertain terms are bounded as follows: $\alpha_1=0.001$ m/s and $\alpha_2=1$ N. Figs. 2 and 3 show the unmatched and matched uncertainty in this case. Fig. 4 show the sliding variable and in Fig. 5 the phase plane of system is plotted. As shown in these figures, sliding variable reaches to sliding surface $S=0$ in desired time. Also with varying the value of parameter η_2, we are able to adjust the time of stabilizing sliding surface in reaching mode sliding variable. Variation of parameter η_1 leads to changing sliding surface and initial value of sliding variable.
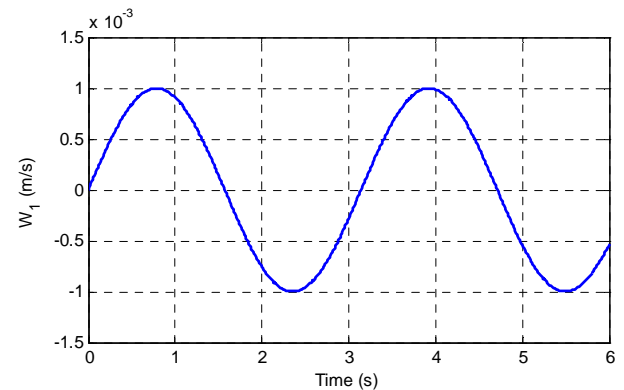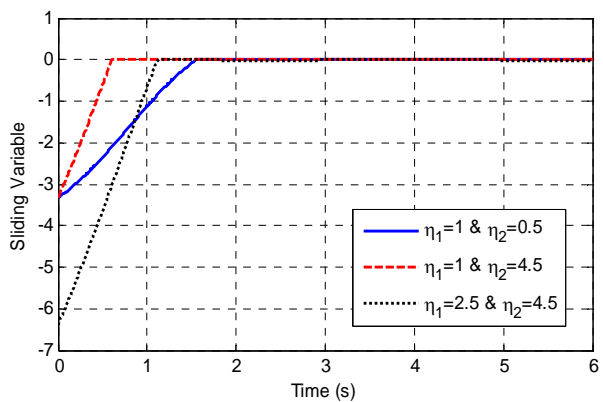

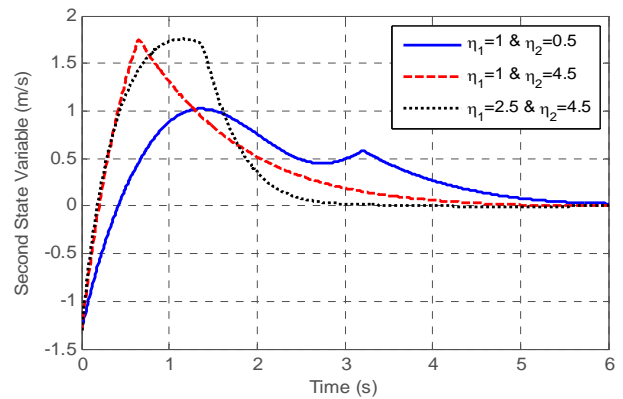**Fig. 2** Unmatched uncertainty (Velocity Variation) in Case I.
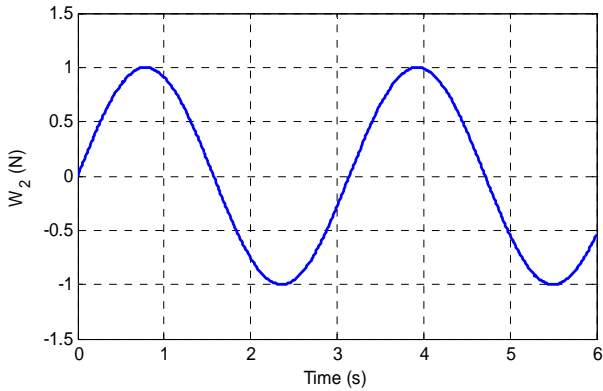
Fig. 3 Matched uncertainty (load force given by the wind) in Case I.
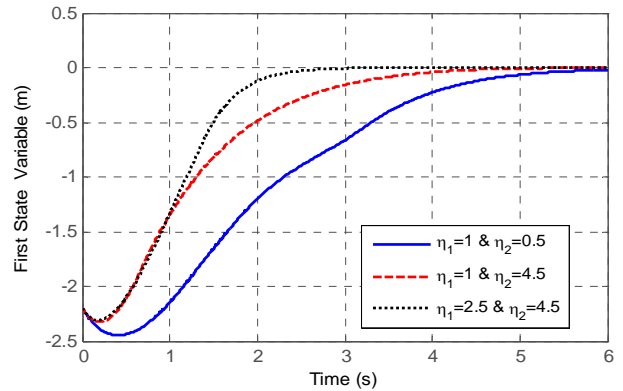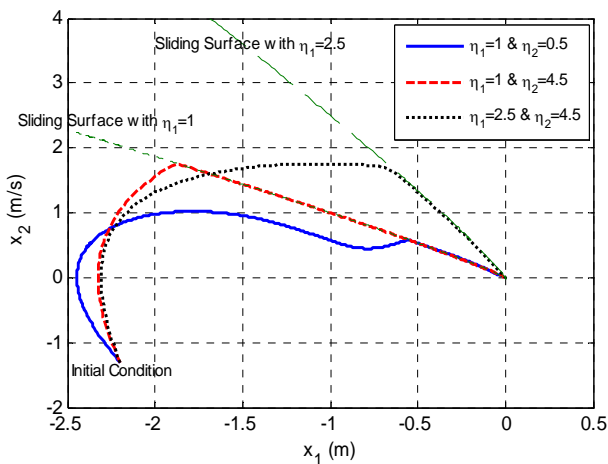

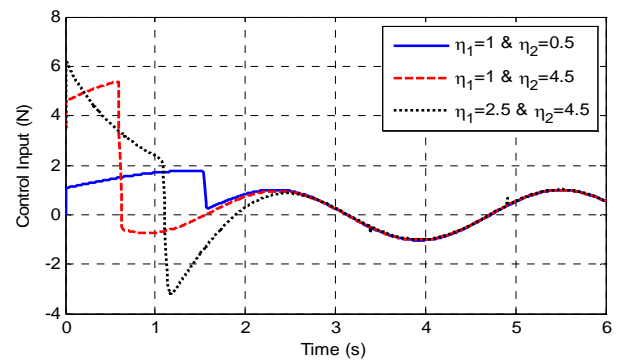**Fig. 6** State variable 1 (x_1) in Case I.

Fig. 4 Sliding variable.

Fig. 7 State variable 2 (x_2) in Case I

Fig. 5 Phase plane in Case I.

Fig. 8 Control input in Case I

Figs. 6 and 7 show the displacement of cart (x_1) and velocity (x_2) with different value for η_1 and η_2. As seen in these figures, the time of zeroing state variables is decreased with increasing the value of η_2. Therefore the stabilization time of state variables is adjustable with parameter η_2 in sliding phase.

Figure 8 present the control input. With increasing the values of η_1 and η_2, the maximum magnitude of the control input is increased. Therefore the terminal sliding mode controller in Eq. (38) is able to stabilizing states and sliding variable in desired finite time.

4.2 Case II: Comparison with Conventional TSMC

In this case, the performance of proposed algorithm is compared with conventional TSMC algorithm. Note that the conventional TSMC is not able to controlling unmatched uncertain systems. This controller is designed with Eq. (9) as sliding variable. Figs. 9 and 10

show the unmatched and matched uncertainty in this case. Therefore the uncertain terms are bounded as follows: α_1=0.5 m/s and α_2=1 N.

Fig. 11 shows the sliding variable. As shown in this figure, sliding variable reaches to sliding surface S=0 in desired time in both proposed and conventional TSMCs in the presence of matched uncertainty (w_2). Fig. 12 shows the displacement of cart (x_1). As seen in this figure, the proposed algorithm is able to controlling this variable in the presence of unmatched uncertainty (w_2) and in the conventional TSMC, this variable is not controlled. Fig. 13 show the second state variable and control signal is plotted in Fig. 14.
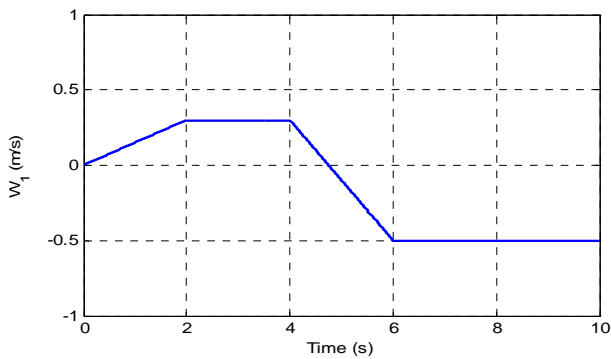

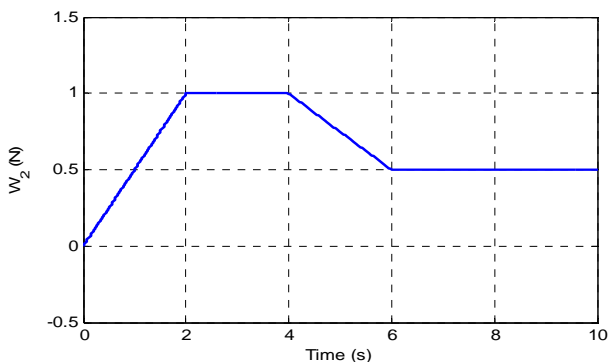**Fig. 9** Unmatched uncertainty (Velocity Variation) in Case II.

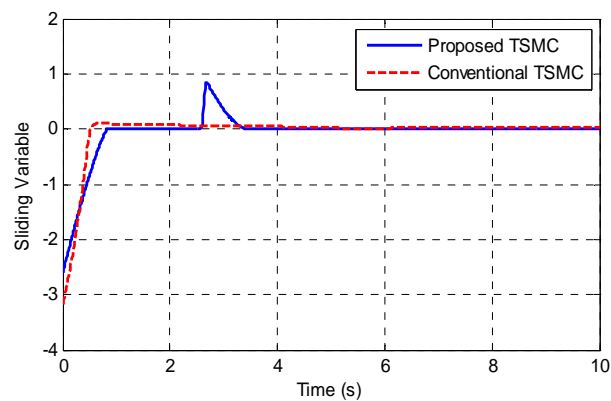**Fig. 10** Matched uncertainty (load force given by the wind) in Case II.
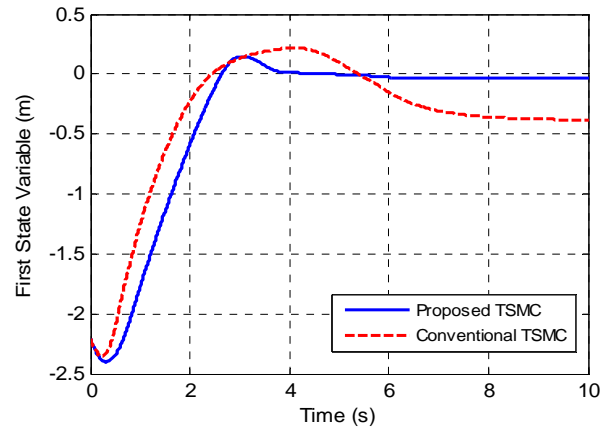

Fig. 11 Sliding variable in Case II.

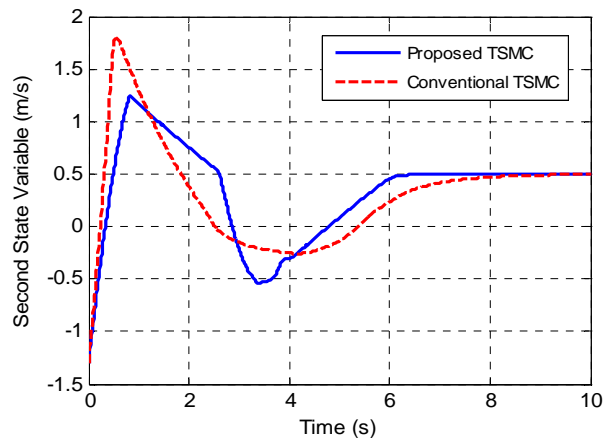**Fig. 12** State variable 1 (x_1) in Case II.

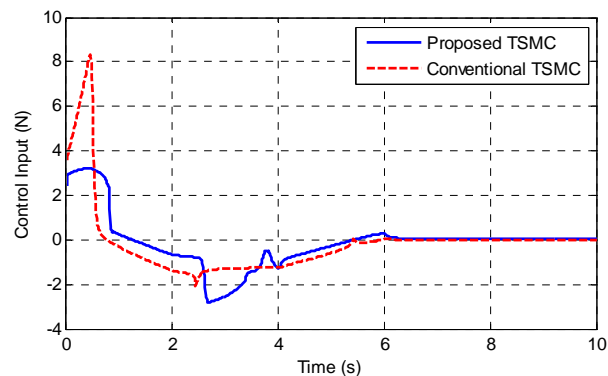**Fig. 13** State variable 2 (x_2) in Case II.

Fig. 14 Control input in Case II.

5 Conclusion

In this paper, a new TSMC is proposed. This algorithm leads to improving the overall performance of control system by applying finite time stability condition and without estimation of matched and unmatched uncertainty. The proposed controller guarantees the convergence of the states and sliding variables to zero in desired finite time. Simulation results show the effectiveness and robustness of the controller in control of systems with both matched and unmatched uncertainties.

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