General Formulation to Investigate Scattering from Multilayer Lossy Inhomogeneous Metamaterial Planar Structures

M. Kiani*, A. Abdolali*

Abstract: This paper presents a general formulation to investigate the scattering from Multilayer Lossy Inhomogeneous Metamaterial Planar Structure (MLIMPS) with arbitrary number of layers and polarizations. First, the dominating differential equations of transverse components of electromagnetic fields in each layer are derived. Considering the general solution form of the differential equations and the boundary conditions of the problem; a set of linear equations is obtained. By solving these equations, the electromagnetic fields in all the layers and reflection and transmission coefficients are calculated. This method is used in analyzing two inhomogeneous planar layers consisting of conventional material and metamaterial in an interesting example. The presented results are useful for constructing general duality between conventional materials and metamaterials.

Keywords: Duality, General formulation, Multilayer Lossy Inhomogeneous Metamaterial Planar Structure (MLIMPS).

1 Introduction

Homogeneous planar structures are the most applicable structures in filters, shielding and etc. Inhomogeneous materials in these structures can optimize shielding effectiveness and provide less scattering and larger bandwidth [1-10]. There are different methods to analysis an inhomogeneous planar multilayer structure such as Taylor's series expansion [11], Fourier's series expansion [12], Equivalent source method [13], Method of moment [14], Inverse scattering method [15] and so on. All these methods are numerical and suggested for a slab composed of inhomogeneous medium. In this paper, a general formulation for investigating scattering from arbitrary Multilayer Lossy Inhomogeneous Planar Structure (MLIMPS) with an arbitrary number of layers and polarizations is presented. Dominating differential equations of MLIMPS are written in form of a linear second order differential equation with non-constant coefficient. General solution form of these equations is presented. Using boundary conditions of the problem, a set of linear equations is obtained which leads to calculating the electromagnetic fields in all layers and reflection and transmission coefficients. This formulation is applicable in stating general fundamentals and properties of multilayer

inhomogeneous metamaterial structures and can be used for providing conditions for intended phenomena for the mentioned structures such as zero reflection, zero transmission, cloaking and etc. Finally, the presented method is applied in an interesting example that is useful for establishing a duality between conventional materials and metamaterials in general case [16].

2 Differential Equation of MLIMPS

Fig. 1 indicates a typical MLIMPS with N layers. The left half space is free space and the right one is an arbitrary homogeneous medium. Constitutive parameters of kth layer are as follow:

$$\mu_k(z) = \mu_{Rk}(z) - j\mu_{Ik}(z) \tag{1}$$

$$\varepsilon_k(z) = \varepsilon_{Rk}(z) - j\varepsilon_{Ik}(z) \tag{2}$$

for $k = 1, 2, 3, ..., N$.

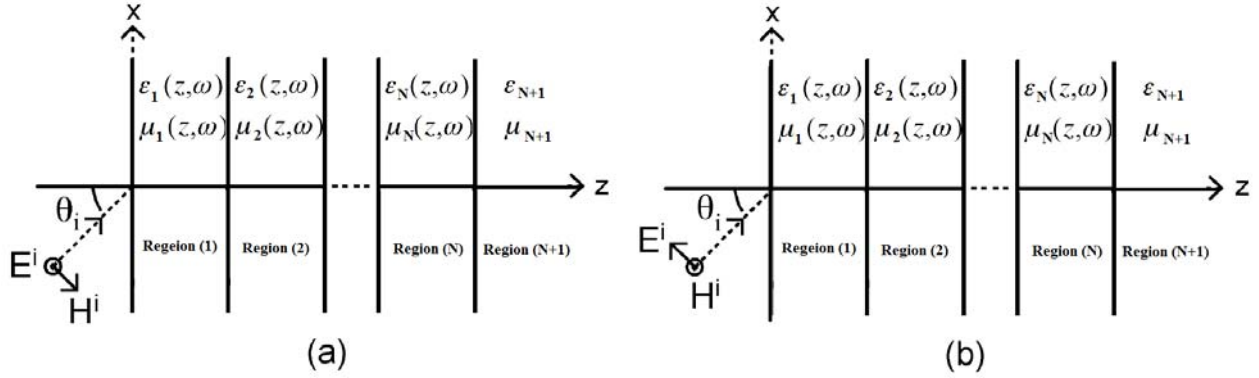By combination of Maxwell's Curl equations, we can write vector wave equations for k^{th} layer as follows [17].

$$\nabla \times (\mu_k^{-1}(z) \times \nabla \times E_k) = \omega^2 \varepsilon_k(z) E_k \tag{3}$$

$$\nabla \times (\varepsilon_k^{-1}(z) \times \nabla \times H_k) = \omega^2 \mu_k(z) H_k \tag{4}$$

Because of the infinite dimension of the structure in y direction, we have:

$$\frac{\partial}{\partial y} = 0 \tag{5}$$

Fig. 1 Typical lossy inhomogeneous planar multilayer structure (a) TE^z (b) TM^z

Under phase-matching condition in the boundary of each two consecutive medium, we have

$$Q(x,z) = P(z)e^{-jK_x x} \quad \text{or} \quad \frac{\partial}{\partial x} = -jK_x \quad (6)$$

where:

$$K_x = K_0 \sin\theta_i \quad (7)$$

$$K_0 = \omega\sqrt{\varepsilon_0\mu_0} \quad (8)$$

The above equations, indicates the general form of electromagnetic field in each medium.

Using Eq. (3, 4) and noticing Eq. (5, 6), differential equation of transverse components of electromagnetic fields in k^{th} medium can be obtained as follows:

For TE^z polarization

$$\frac{d^2 E_{ky}(z)}{dz^2} - \frac{\mu'_k(z)}{\mu_k(z)}\frac{dE_{ky}(z)}{dz} + \quad (9)$$

$$(\omega^2\mu_k(z)\varepsilon_k(z) - K_x^2)E_{ky}(z) = 0$$

where:

$$\mu'_k(z) = \frac{d\mu_k(z)}{dz} \quad (10)$$

For TM^z polarization

$$\frac{d^2 H_{ky}(z)}{dz^2} - \frac{\varepsilon'_k(z)}{\varepsilon_k(z)}\frac{dH_{ky}(z)}{dz} \quad (11)$$

$$+(\omega^2\mu_k(z)\varepsilon_k(z) - K_x^2)H_{ky}(z) = 0$$

where:

$$\varepsilon'_k(z) = \frac{d\varepsilon_k(z)}{dz} \quad (12)$$

where:

$$\begin{cases} E_{ky}(x,z) = E_{ky}(z)e^{-jK_x x} & \text{TE} \\ H_{ky}(x,z) = H_{ky}(z)e^{-jK_x x} & \text{TM} \end{cases} \quad (13)$$

Eqs. (9) and (11) are second order linear differential equations with non-constant coefficients. From principles of ordinary differential equations, second order linear differential equation of transverse fields in k^{th} inhomogeneous medium has a general solution as follows:

$$C_{k,1}f_k(z) + C_{k,2}g_k(z) = \begin{cases} E_{ky}(z) & \text{TE} \\ H_{ky}(z) & \text{TM} \end{cases} \quad (14)$$

$$k = 1,2,......,N$$

In Eq. (14), f_k and g_k show the variation of transverse electromagnetic fields in k^{th} layer and $C_{k,1}$ and $C_{k,2}$ are unknown constant coefficients which can be calculated by using the boundary conditions named matching of tangential components of fields at the interface between k^{th} and $k+1^{th}$ layer.

2.2 Boundary Conditions

Considering the general form of transverse fields in Eq. (14), the boundary conditions for tangential component of electric and magnetic fields in each consecutive layer are written as follows:

At the interface $z = 0$

$$F^i_t(z)\big|_{z=0} + F^r_t(z)\big|_{z=0} = (C_{1,1}f_1(z) + C_{1,2}g_1(z))\big|_{z=0} \quad (15)$$

$$\frac{F^i_t(z) - F^r_t(z)\big|_{z=0}}{I_S} = \quad (16)$$

$$\frac{j}{\omega p_1(z)}(C_{1,1}f'_1(z) + C_{1,2}g'_1(z))\big|_{z=0}$$

At the interface $z = d_k$ $(k = 1, 2, ..., N-1)$

$$(C_{k,1}f_k(z) + C_{k,2}g_k(z))\Big|_{z=d_k} = \qquad (17)$$

$$C_{k+1,1}f_{k+1}(z) + C_{k+1,2}g_{k+1}(z)\Big|_{z=d_k}$$

and

$$[\frac{1}{p_k(z)}(C_{k,1}f_k{}'(z) + C_{k,2}g_k{}'(z))]\Big|_{z=d_k}$$

$$= [\frac{1}{p_k(z)}C_{k+1,1}f_{k+1}{}'(z) + C_{k+1,2}g_k{}'(z)]\Big|_{z=d_k} \qquad (18)$$

Furthermore, at the interface $z = d_N$

$$C_{N,1}f_N(z) + C_{N,2}g_N(z)\Big|_{z=d_N} = F^T{}_t(z)\Big|_{z=d_N} \qquad (19)$$

and

$$[\frac{j}{\omega p_N(z)}(C_{N,1}f_N{}'(z) + C_{N,2}g_N{}'(z))]\Big|_{z=d_N} = \frac{F^T{}_t(z)\Big|_{z=d_N}}{I_L} \qquad (20)$$

In Eqs (15)-(20), we have the following definitions:

$$I_S = \begin{cases} \dfrac{\eta_0}{\cos\theta_i} & \text{TE} \\[2ex] \dfrac{1}{\eta_0\cos\theta_i} & \text{TM} \end{cases} \qquad (21)$$

$$I_L = \begin{cases} \dfrac{\eta_1}{\cos\theta_t} & \text{TE} \\[2ex] \dfrac{1}{\eta_1\cos\theta_t} & \text{TM} \end{cases} \qquad (22)$$

$$p_k(z) = \begin{cases} \mu_k(z) & \text{TE} \\ \varepsilon_k(z) & \text{TM} \end{cases} \qquad (23)$$

$$F^i{}_t(z) = \begin{cases} E^i{}_y(z) & \text{TE} \\ H^i{}_y(z) & \text{TM} \end{cases} \qquad (24)$$

$$F^r{}_t(z) = \begin{cases} E^r{}_y(z) & \text{TE} \\ H^r{}_y(z) & \text{TM} \end{cases} \qquad (25)$$

$$F^T{}_t(z) = \begin{cases} E^T{}_y(z) & \text{TE} \\ H^T{}_y(z) & \text{TM} \end{cases} \qquad (26)$$

$$F^i{}_t(x, z) = F^i{}_t(z)e^{-jK_x x} \qquad (27)$$

$$F^r{}_t(x, z) = F^r{}_t(z)e^{-jK_x x} \qquad (28)$$

$$F^T{}_t(x, z) = F^T{}_t(z)e^{-jK_x x} \qquad (29)$$

Reflection and transmission coefficients are defined as follows:

$$\Gamma = \frac{F^r{}_t(0)}{F^i{}_t(0)} \qquad (30)$$

and

$$T = \frac{F^T{}_t(d_N)}{F^i{}_t(0)} \qquad (31)$$

By substituting the Eqs. (30, 31) in Eqs. (15-29), 2N+2 linear equations with 2N+2 unknown variables are derived which can be solved easily to obtain reflection and transmission coefficients and electromagnetic fields in all regions. These unknown variables can be calculated as follows:

$$\begin{aligned} &C_{k,1}, C_{k,2} \\ &1 \le k \le N \\ &\Gamma, T \end{aligned} \qquad (32)$$

Therefore, a set of linear equations can be written as a suitable matrix of equations in the form of Eq. (33). This matrix of equations is applicable and so useful in analyzing the arbitrary type of MLIMPS and this formulation can be introduced as a full-wave method for the analysis of MILPS. As a potential application, this method can lead researchers to interesting phenomena such as zero reflection and zero transmission and can construct a novel duality between conventional materials and metamaterials in general case.

$$\begin{bmatrix}
-\dot{H}^i(0) & f_1(z)\big|_{z=0} & g_1(z)\big|_{z=0} & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
\dfrac{\dot{H}^i(0)}{I_S} & \dfrac{jf_1'(z)}{\omega p_1(z)}\big|_{z=0} & \dfrac{jg_1'(z)}{\omega p_1(z)}\big|_{z=0} & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
0 & & & & \vdots & & \vdots & & & \vdots & & \\
0 & 0 & 0 & \cdots & f_k(z)\big|_{z=d_k} & g_k(z)\big|_{z=d_k} & -f_{k+1}(z)\big|_{z=d_k} & -g_{k+1}(z)\big|_{z=d_k} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \cdots & \dfrac{f_k'(z)}{p_k(z)}\big|_{z=d_k} & \dfrac{g_k'(z)}{p_k(z)}\big|_{z=d_k} & \dfrac{-f_{k+1}'(z)}{p_{k+1}(z)}\big|_{z=d_k} & \dfrac{-g_{k+1}'(z)}{p_{k+1}(z)}\big|_{z=d_k} & 0 & 0 & 0 & 0 \\
0 & & & & & \vdots & & & & & & \\
0 & & & & 0 & \cdots & 0 & 0 & f_N(z)\big|_{z=d_N} & g_N(z)\big|_{z=d_N} & -\dot{H}^i(0) & \\
0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \dfrac{jf_N'(z)}{\omega p_N(z)}\big|_{z=d_N} & \dfrac{jg_N'(z)}{\omega p_N(z)}\big|_{z=d_N} & \dfrac{-\dot{H}^i(0)}{I_L} &
\end{bmatrix}
\begin{bmatrix} \Gamma \\ C_{1,1} \\ C_{1,2} \\ \vdots \\ C_{k,1} \\ C_{k,2} \\ C_{k+1,1} \\ C_{k+1,2} \\ \vdots \\ C_{N1} \\ C_{N2} \\ T \end{bmatrix}
=
\begin{bmatrix} \dot{H}^i(0) \\ \dfrac{\dot{H}^i(0)}{I_S} \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \end{bmatrix}
\quad (33)$$

3 Example and Discussion

In this section, scattering of electromagnetic waves from two planar inhomogeneous structures composed of conventional material and metamaterial, situated between two free half spaces, are studied through an interesting example. Electromagnetic parameters of inhomogeneous layers are as shown in Table 1, where '*' denotes the complex-conjugate operator. It is assumed that $\varepsilon_{r0} = 4$, K is an arbitrary complex constant and thickness of each inhomogeneous layer is $d = 20\,\text{cm}$. A TE^z polarized plane wave with the electric field strength of $E^i = 1.0$ V/m and the excitation frequency of 1.0 GHz is illuminated to the assumed structures obliquely. According to relations purposed in [11-15], we can write general form of electric fields through Bessel functions $J_v(x)$ as follows:

$$E_{y1}(x,z) = e^{-jK_x x}$$
$$[A_1 J_{\left(\frac{2K_x}{K}\right)}\left(\frac{2K_0\sqrt{\varepsilon_{r0}}}{K}\exp(\frac{K}{2}z)\right)$$
$$+B_1 J_{\left(-\frac{2K_x}{K}\right)}\left(\frac{2K_0\sqrt{\varepsilon_{r0}}}{K}\exp(\frac{K}{2}z)\right)] \quad (34)$$

Table 1 Constitutive parameters of the structures under consideration.

	Structure 1	Structure 2
Relative permeability	$\mu_r = 1$	$\varepsilon_r = \varepsilon_{r0}e^{Kz}$
Relative permittivity	$\mu_r = -1$	$\varepsilon_r = -\varepsilon_{r0}e^{K^* z}$

$$E_{y2}(x,z) = e^{-jK_x x}$$
$$[A_2 J_{\left(-\frac{2K_x}{K^*}\right)}\left(\frac{2K_0\sqrt{-\varepsilon_{r0}}}{K^*}\exp(\frac{-K^*}{2}z)\right)$$
$$+B_2 J_{\left(\frac{2K_x}{K^*}\right)}\left(+\frac{2K_0\sqrt{-\varepsilon_{r0}}}{K^*}\exp(\frac{-K^*}{2}z)\right)] \quad (35)$$

According to differential equations concepts and Eq. (9) and the rules mentioned in [13] it can be easily concluded that considering electromagnetic parameters in these inhomogeneous slabs being negative conjugates of one another, general forms of variations in z direction in these structures will be conjugates of each other. Figs. 2-3 represent amplitudes and imaginary parts of transverse electric fields in the two structures at angle of incidence $\theta_i = 60^\circ$ for $K = 1$ and $K = 5$. The purposed curves show great agreement with the results in [11-14].

Amplitudes and imaginary parts of reflection coefficients of the two structures at the angle of incidence between 0 and 90 for $K = 1$ and $K = 5$ are shown in Figs. 4-5. The results show great agreement with the curves purposed in [11-14].

As it can be seen from the purposed relations for the fields, they are not complex-conjugates of each other due to presence of exponential term $e^{-jK_x x}$ whereas reflection coefficients are conjugates of one another. It can be interpreted according to the concepts purposed in [16]. If we model inhomogeneous slabs of the two structures by two port networks and transfer matrixes as it is shown in Fig. (6), components of transfer matrices of these networks are conjugates of one another. Thus
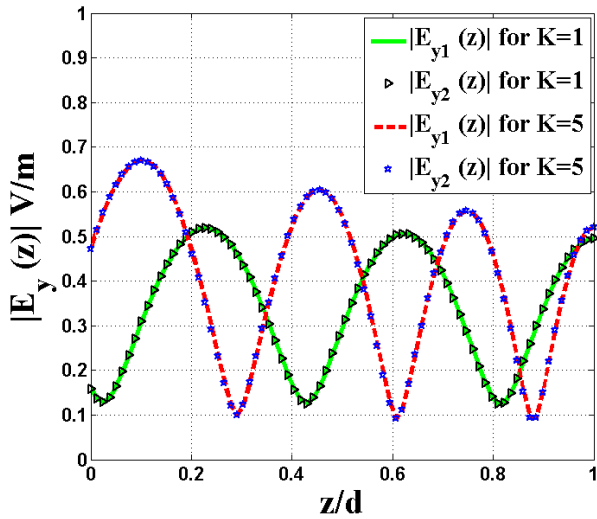
Fig. 2 Amplitudes of the transverse components of the electric fields related to structure 1 and structure 2
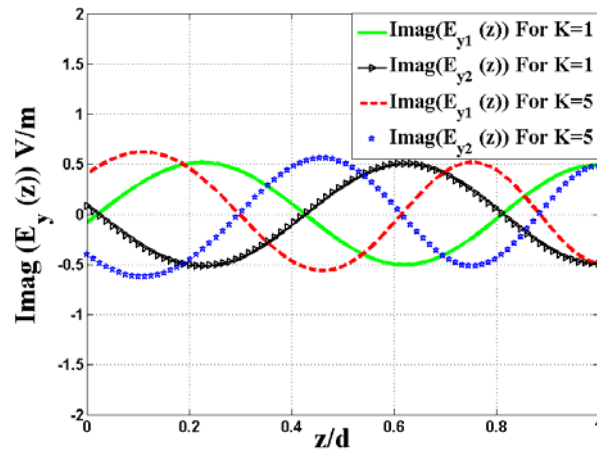

Fig. 4 Amplitude of the reflection coefficients for structure 1 and structure 2.

Fig. 3 Imaginary parts of the transverse components of the electric fields for structure 1 and structure 2
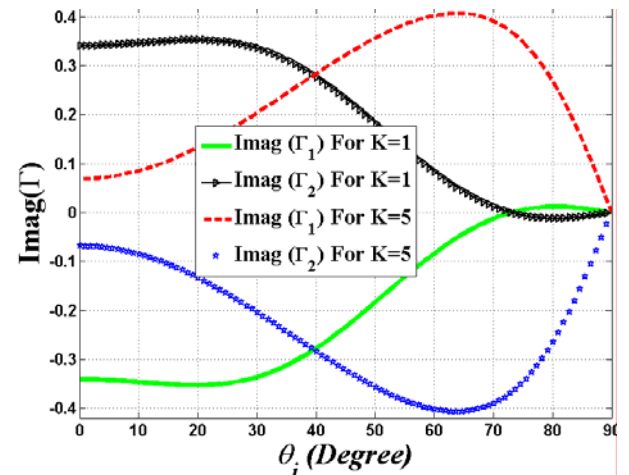
Fig. 5 Imaginary part of the reflection coefficients for structure 1 and structure 2

$$T_1 = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \qquad (36)$$

$$T_2 = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} = \begin{bmatrix} A_1^* & B_1^* \\ C_1^* & D_1^* \end{bmatrix} \qquad (37)$$

The impedances of source and load indicate the media of two half spaces. Since inhomogeneous slabs are situated between two half spaces composed of lossless media and according to the second theorem purposed in [16], considering definition of input impedance and reflection coefficients for two port networks, reflection coefficients of the two structures are conjugates of one another. Thus
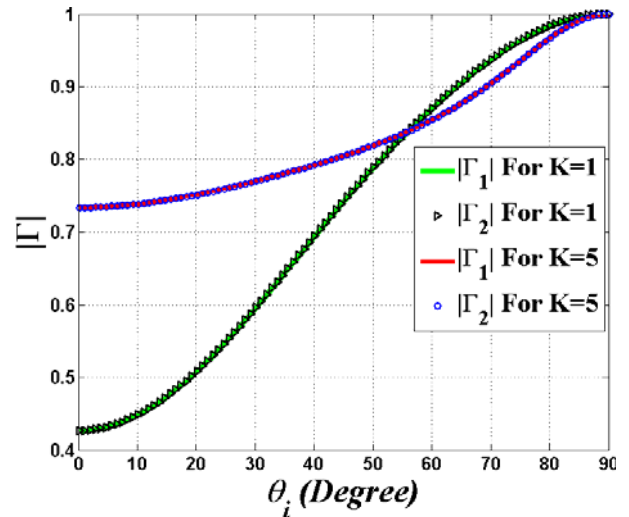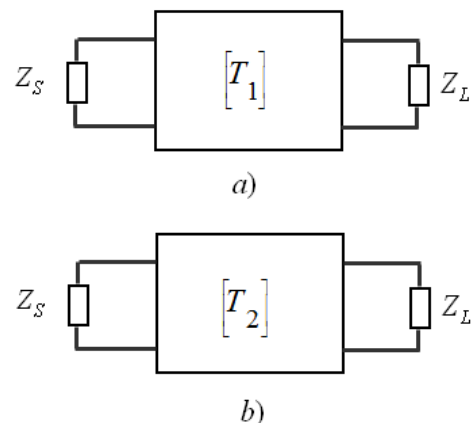

Fig. 6 A two-port network model for inhomogeneous planar layer a) Structure 1 b) Structure 2

$$Z_{in}{}^1 = \frac{A_1 Z_L + B_1}{C_1 Z_L + D_1} \tag{38}$$

$$Z_{in}{}^2 = \frac{A_2 Z_L + B_2}{C_2 Z_L + D_2} \tag{39}$$

$$\Gamma_1 = \frac{Z_{in}{}^1 - Z_L}{Z_{in}{}^1 + Z_L} \tag{40}$$

$$\Gamma_2 = \frac{Z_{in}{}^2 - Z_L}{Z_{in}{}^2 + Z_L} \tag{41}$$

Presented results can be seen as an initiative by researchers for establishing a duality between conventional and metamaterial media in much more general case than it is proved in [16].

5 Conclusion

General formulation to investigate the scattering from MLIMPS was presented. Using Maxwell's equations and noticing physical and geometrical properties of the structure and plane wave which illuminate the assumed structure, dominating differential equations of MLIMPS were derived. From principles of ordinary differential equations, the general form of solution of wave equation for transverse components of electric and magnetic field is presented and with this general solution and using the boundary conditions of the problem, a set of linear equations was obtained. Solving these equations, reflection and transmission coefficient and electromagnetic fields in all regions were calculated. Finally the proposed method was used in an interesting example for two conventional material structure and metamaterial structure which can be useful for establishing general duality between conventional materials and metamaterials.

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