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Technical Note

Comparison of some probabilistic methods for analyzing slope stability problem

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Abstract

The study aims at comparing the results of different probabilistic methods such as the perturbation method, Spectral Stochastic Finite Element Method (SSFEM) and Monte Carlo Method. These methods are developed for a linear soil behavior. In this study, by assuming soil strength properties and Young modulus are random variables, a preliminary search of critical slip surface is established. One-dimensional random field is used to conduct a parametric study. The proposed probabilistic slope stability analysis is performed using a performance function formulated by the stochastic stress field mobilized along the slip surface. The results have shown that the Young modulus has no significant effect on the factor of safety. In addition, the studies have found that the perturbation method is valid than SSFEM to conduct the slope stability analysis. Moreover, SSFEM performed with high orders of expansion to reach the convergence of solution can lead to intractable calculations, while Monte Carlo method is too time consuming for a slope stability reliability analysis using modeling the spatial variability of soil properties by the random field's theory. Finally, the numerical results have shown that the correlation lengths of the soil strength properties have effect on the position of the critical sliding surface.

Keywords: *Slope stability, Stochastic finite element method, Random field, Probabilistic methods, Monte Carlo.*

1. Introduction

Uncertainty and hazard are unavoidable in the geotechnical engineering domain. The adopted analytical models are mathematical simplifications of more complex physical phenomena, and soil parameters introduced in some equations present a spatial variability. For many years, the analysis of slope stability has been studied by deterministic approaches that have considered these uncertainties by determining a safety factor. One such example is the finite element method.

Since risks have begun to be quantified and discussed, probabilistic methods are now required to overcome this deficiency. Researchers quantify the uncertainty in the soil properties, either by random variable or by the random field's theory. In recent years, several probabilistic approaches have been developed for the analysis of slope stability [1, 2,3,4]. In considering, the spatial variability of soil properties, the random fields can be considered as an appropriate tool for reliability analysis in the geotechnical engineering.

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Most methods can take into account the spatial variability of soil properties that are modeled by the random field's theory. The random field is completely defined by a mean, standard deviation and autocorrelation function.

The work presented here aims to compare the perturbation method, spectral stochastic finite element method SSFEM and Monte Carlo simulation to calculate the statistical parameters of safety factors for a probabilistic slope stability analysis of homogeneous soil within the linear behavior law. A discussion of the applicability and effectiveness of these methods is presented for evaluation of probability density safety factor as well as its mean and standard deviation. The discretization procedure is necessary to approximate with an estimation error and is divided into three major types: point discritization, average discritization and series expansion methods [5, 6, 7].

The probabilistic methods proposed in this article, which belong to the stochastic finite element method, are applied to various domains by different formulations. Among them are the perturbation method and the spectral method [7, 8, 9, 10, 11]. However, most of these formulations are used for linear constitutive law. There are only a few documents that study the reliability problem with elastoplastic material behavior [9, 10, 11,12, 13]. Similar works with convergence study and the evaluation of error were published by Babuska and Chatzipantelidis [14]. In reliability context, a comprehensive parametric study is conducted to investigate the influence the young modulus on the factor of safety. A discussion of the effectiveness of these approaches has been illustrated here through numerical examples meant to analyze slope stability by taking into account the spatial variability of the given soil properties.

2. Theoretical Background

2.1. The performance function for slope stability analysis

In the proposed probabilistic analysis, the performance function G is formulated using the factors of safety.

The performance function is given by the following relation:

$$G(\xi) = F_S - 1 \quad (1)$$

For a circular slip surface divided on n elements, the expression of the factor of safety is:

$$F_S = \frac{\sum_{i=1}^{n} \left(c + \sigma_i \tan(\varphi)\right)\Delta l_i}{\sum_{i=1}^{n} \tau_i \Delta l_i} \quad (2)$$

In this equation: n is the number of elements in the slip surface; c is cohesion; φ is the internal friction angle; σ_i is the normal stress; and τ is the shear stress. For a segment of the slip surface of length Δl_i located in the ith element, inclined at an angle α to the horizontal plan.

2.2. The used methods for slope stability analysis

In this paper, Young's modulus, cohesion and friction angle are assumed random fields. The Karhunen-loève expansion [15] of the random field based on the spectral decomposition of its auto-covariance function is used. Additionally, the perturbation method, Spectral Stochastic Finite Element Method (SSFEM) and Monte Carlo Method are developed for an elastic soil behavior.

2.2.1. Perturbation methods with spatially varying soil proprieties

The perturbation method uses a Taylor series expansion about the mean values of random functions for quasi-static linear problems.

Using a Taylor expansion at second order with respect N variable ξ_i, the solution of the equilibrium equation:

$$KU = F \quad (3)$$

is given by:

$$U_O = K_O^{-1} F_O \quad (4)$$

$$U_i^I = K_O^{-1}\left(F_i^I - K_i^I U_O\right) \quad (5)$$

$$U_i^{II} = K_O^{-1}\left(F_{ij}^{II} - K_i^I U_i^I - K_{ij}^{II} U_O\right) \quad (6)$$

Where: K is the stiffness matrix, U and F are respectively the displacement and load.

The first $()_i^I$and second $()_{ij}^{II}$order coefficients are obtained from the derivatives of the corresponding quantities evaluated at $\xi = 0$. K_0 , U_0 and F_O take for these mean values of the input

2.2.2. Spectral stochastic finite element method SSFEM

The Spectral Stochastic Finite Element Method (SSFEM) developed by Ganem and Spanos in 1991[16] is based on a decomposition of random fields over a space basis proposed by Karhunen-Loeve [15].In addition, decomposition of the nodal displacement vector solution of the problem over polynomial chaos basis ψ is used.

To determine the unknown of random vectors, the stuffiness matrix is truncated at N+1 terms, and at P terms for the displacements vector expansion:

$$U(\theta) = \sum_{k=0}^{P-1} U_k \psi_k(\{\xi_k\}_{k=1}^{N}) \quad (7)$$

Where: U_k deterministic vectors having M (The number of degrees of freedom) components; P is the number of polynomial chaos with order less than or equal to p (p: order of polynomial chaos) and N (N: order of the random field expansion).

The stochastic stress is approximated by the following relation:

$$\sigma(x,\theta) = E(x,\theta) D_0 B(x) \sum_{j=1}^{P-1} u_j^e \psi_k\left(\{\xi_k(\theta)\}_{k=1}^{N}\right) \quad (8)$$

Where: u_j^e are the displacement vector components of the element in the polynomial chaos basis. D_0 Constant elasticity matrix depends to the Poisson ratio, $B(x)$ matrix that relates the components of stain to the element nodal displacement. $E(x,\theta)$: The Young modulus modeled by the random filed.

The Finite Element Reliability Using Matlab (FERUM) program developed in 2001 by Haukaas and Der Kiureghian at the university of California at Berkeley coupled with an additional program written in Matlab was used for slope stability analysis [17].

2.2.3. Monte carlo simulation

The idea behind using a Monte Carlo simulation was to simulate a number of realizations of the random field. For each realization the response quantity was computed. The sample corresponds to the generation of N independent standard normal variables according to the Karhunen-loève expansion truncated at N terms. The response of a system was evaluated by a deterministic finite element code with their statistical treatment subsequently performed.

3. Case Study

3.1. The purpose of the study

In order to study the different probabilistic methods, slope with elastic soil behavior has been considered. The Young's modulus and the soil strength parameters c and φ have been regarded as one-dimensional random field along the vertical direction. A slope with no foundation layer is used to carry out a preliminary search of critical slip surface assuming strength soil parameters and Young modulus random variables. In addition, one dimensional random field along the depth is used to conduct a parametric study. An exponential autocorrelation function was adopted. The random fields are regarded as Gaussian random fields, completely defined by their means, variances and autocorrelation functions. The critical deterministic slip surface corresponds to minimum factor of safety among all examined geometrically possible circular sliding surfaces, while the critical probabilistic sliding surface corresponds to the minimum reliability index. The mean and standard deviation of the factor of safety are evaluated for the probabilistic circular slip surface.

3.2. Numerical results

The problem geometry, the slope's mesh constituted by quadrangle elements (QUAD4) and the boundary conditions are indicated in the Fig. 1. The mean and coefficients of variation are specified in Table 1. A plane strain analysis is carried out.

Table 1 Inputs soil properties

Soil properties	μ	cov
c(kPa)	7	0.2
φ(°)	27	0.2
$\gamma(KN/m^3)$	19	---
E(MPa)	15	0.2
υ	0.25	---

c: Cohesion. φ: Friction angle. γ: Unit weight. E: Modulus Young.:υ Poisson ratio
μ Mean. cov: coefficient of variation.

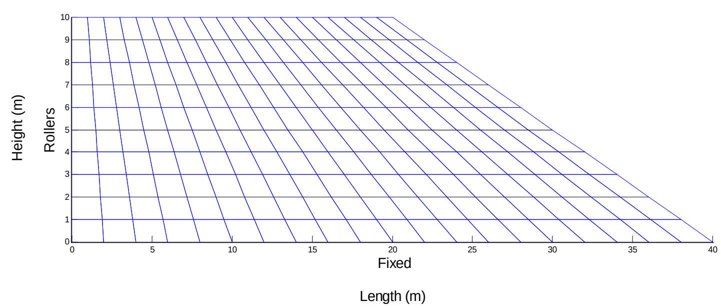

Fig. 1 Meshing of the slope

The critical probabilistic surface is determined, by admitting that the soil strength parameters and the Young's modulus are uncorrelated random variables in the attention to obtain conservative results [18]. It is used to carry out the homogeneous slope stability analysis using the random field's theory. In Table 2. are reported the geometrical characteristics of the critical slip surfaces, the reliability indices estimated by HL-RF Algorithm [19] and the corresponding factors of safety. These results are evaluated by the deterministic and probabilistic finite element methods using the expression of the factor of safety given by the equation Eq (2).

Table 2 Results of the deterministic and probabilistic approach neglecting the spatial variability

		Deterministic FE	Probabilistic FE
Factor of safety F_S		1.631	1.652
Reliability index β		2.462	2.3891
The geometric characteristics of slip surface	R(m)	31.597	33.703
	x_c(m)	40.000	40.600
	y_c(m)	31.602	33.702

R: radius
Circle center: (x_c, y_c)
FE: Finite elements.

In order to obtain a comprehensive study of the results, a parametric analysis is carried out to discuss the influence of Young modulus on the factor of safety. For different coefficients of variation of Young modulus, random field's correlation lengths and the expansion order are set equal to $l = 5m$, N=3 respectively, the means and standard deviations of the safety factors are estimated. The number of Monte Carlo simulation used which ensures convergence is 10,000. The SSFEM is performed for N=3 and p=3 The results of the methods are listed in Table 3.

Table 3 Influence of the Young modulus coefficient of variation

Coefficient of variation of the Young modulus cov_E	Perturbation M. 2ieme Order		SSFE M.		Monte Carlo M.	
	μ_F	σ_F	μ_F	σ_F	μ_F	σ_F
0.10	1.621	0.202	1.440	0.199	1.239	0.067
0.20	1.622	0.202	1.445	0.203	1.227	0.062
0.30	1.593	0.202	1.454	0.214	1.230	0.067
0.40	1.624	0.202	1.562	0.249	1.210	0.056
0.50	1.597	0.202	1.706	0.325	1.223	0.050

For the purpose to fully investigate the influence the Young modulus correlation length, we have been evaluated the means and standard deviation of the factors of safety for different correlation lengths. The coefficients of variation of the random fields are set equal to 0.2. In Table 4, we reported the results of the different methods.

By adopting the values listed in Table 1 , Young modulus, friction angle and cohesion are assumed one-dimensional random fields along the depth and its expansion order equal to N=4, a search for probabilistic slip surface is established by the perturbation method presented previously for the correlation lengths equal to $l = 5$ and $l = 50$. The results are listed in the Table 5.

Table 4 Influence of the Young modulus correlation length

Correlation length of E l(m)	Perturbation M. 2ieme Order		SSFE M.		Monte Carlo M.	
	μ_F	σ_F	μ_F	σ_F	μ_F	σ_F
5	1.622	0.202	1.445	0.203	1.227	0.062
10	1.608	0.202	1.448	0.207	1.242	0.040
20	1.610	0.202	1.448	0.207	1.219	0.075
30	1.616	0.202	1.448	0.207	1.226	0.075
40	1.602	0.202	1.448	0.207	1.240	0.079
50	1.622	0.202	1.448	0.207	1.236	0.089

Table 5 Probabilistic slip surfaces

		Perturbation M. 2ieme Order Correlation length	
		$l = 5\ m$	$l = 50\ m$
Reliability index β		3.238	2.4885
The geometric characteristics of slip surfaces	R(m)	32.199	35.503
	x_C(m)	40.000	40.600
	y_c(m)	33.200	35.500

4. Discussions

According to the analysis established in this paper, the following remarks can be given:

- In considering the distinct positions of the critical slip surfaces, the difference values of the reliability indices and the safety factors, listed in Table 2, the probabilistic and deterministic study of the slope stability analysis can be considered complementary. This result highlights the interest of the stochastic methods to analyze the stability of the geotechnical designs.
- The comprehensive parametric study carried out to determine the effect of Young modulus on the factor of safety revealed that it has not a significant effect to the mean and standard deviation of the safety factor. This result is validated by the proposed methods. It is noted that several established studies carried out using the deterministic finite element method to evaluate the safety factor have made it clear that it depends mainly on the distribution of the stress field [20].
- The perturbation method is valid to conduct the slope stability analysis compared to the SSFEM method that requires a high expansion order to reach the convergence of solutions. Moreover, the perturbation method and SSFEM method give comparable values of the

standards deviations of the safety factor

- The Monte Carlo simulation carried out with Gaussian random fields has under estimated the mean and standard deviation of the safety factor compared to other methods. Despite that it is too time consuming; it remains the most frequently used tool for a slope stability reliability analysis using modeling the spatial variability of soil properties by the random field’s theory.

The results presented in Table 2 and Table 5 shows that taking into account spatial variability over-estimates the reliability index for relatively small coefficient of variation (cov=0.2). The comparison of the obtained probabilistic sliding surface by the perturbation method for different correlation lengths indicates that the research of a probabilistic slip zone of the slope seems more rational than the search of a sliding surface.

5. Conclusion

In this paper, probabilistic methods, proposed for the slope stability analysis, have been used in the context of spatial variability of soil properties. A formulation of each approach has been recalled. It is found that the perturbation method is inexpensive when applied in the case of a Gaussian random field since the second derivative of the stiffness matrix is zero. The convergence and applicability of different approaches were tested with consideration of the variability of the soil proprieties for one- dimensional random fields.

The parametric study has indicated that the Young's modulus does not have a significant effect on the safety factor. In addition, the critical slip surface varies spatially when the spatial variability of soil properties is taken into account. The Monte Carlo simulation has an advantage in that it can obtain a solution with an acceptable error of probability for any problem. However, it can quickly become a time-consuming calculation- especially when the problem is treated with multiple uncertain soil properties. The studies have found that the perturbation method is valid to conduct the slope stability analysis compared to the SSFEM method. Moreover, SSFEM method performed with high orders of expansion lead to intractable calculation. In addition, the perturbation method and SSFEM method give comparable standard deviations values of the factor of safety.

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