Low-amplitude dynamic properties for compacted sand-clay mixtures

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Abstract

Shear modulus and damping ratio are important input parameters in dynamic analysis. A series of resonant column tests was carried out on pure clays and sand-clay mixtures prepared at different densities to investigate the effects of aggregate content, confining stress, void ratio and clay plasticity on the maximum shear modulus and minimum damping ratio. Test results revealed an increase in the maximum shear modulus of the mixture with the increase in sand content up to 60%, followed by a decrease beyond this value. It was also found that the maximum shear modulus increases with confining stress, and decreases with void ratio. In addition, minimum damping ratio increases with sand content and clay plasticity and decreases with confining stress. Finally, on the basis of the test results, a mathematical model was developed for the maximum shear modulus.

1. Introduction

Compacted aggregate-clay mixtures have been successfully used as the cores of embankment dams. These materials, called composite clays by Jafari and Shafiee [1], are usually broadly graded and are composed of clay as the main body with sand, gravel, cobble or even boulders floating in the clay matrix. The Miboro and Ohshirakawa dams in Japan [2], Taguaza dam in Venezuela [3], and Karkheh and Gotvand dams in Iran are some examples of dams with cores composed of aggregate-clay mixtures.

It is also current practice to employ low permeable mixtures of high plastic clay with aggregates as impervious blankets for waste disposal projects [4-8]. It is generally assumed that the coarser portion of such soils imparts a relatively high shear strength, high compacted density and low compressibility while the permeability of the soil is governed by the proportion and nature of the finer portion. This generally results in a relatively serviceable and trouble free fill [9].

A review of the published literature in monotonic loading

reveals that, in general, shear strength either increases with aggregate content or remains constant until a limiting aggregate content, then increases as the aggregate content increases [10-13,1]. On the other hand, the comprehensive studies of Jafari and Shafiee [14,1] showed that in the case of cyclic undrained loading on compacted aggregate-clay mixtures, the assumption that adding aggregate to pure clay improves its mechanical properties is questionable. It was concluded that when aggregate content is raised, cyclic shear strength would decrease. Further numerical investigations by Shafiee [15] and Shafiee et al. [16] revealed that high pore pressure build-up in dams with cores composed of aggregate-clay mixtures may threaten dam stability under seismic loading. soltani-jigheh and soroush [17] and Soroush and soltani-jigheh [18] investigated pre- and post-cyclic behavior of compacted sand-clay mixtures. It was shown that the addition of granular materials to the clayey soils increases excess pore-water pressure during strain-controlled monotonic undrained shearing and cyclic loading. Furthermore, cyclic loading degrade undrained shear strength and deformation modulus of the specimens during the post-cyclic monotonic loading. During post-cyclic monotonic shearing, mixed clayey soils behave similarly to overconsolidated soils; the higher the granular material is, the higher the overconsolidation ratio would be. This shows the need to understand different features of aggregate-clay mixtures behavior particularly under dynamic loading. Determination of low-amplitude dynamic properties of materials is the first step in characterization of dynamic behavior. These properties are

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also essential parameters for ground response and soil-structure interaction analyses.

At very low shear strain levels (less than $10^{-3}\%$), shear modulus (G), and damping ratio (D) remain essentially constant; shear modulus is at its maximum value, and damping ratio at its minimum value. Although it appears that minimum damping ratio (D_{min}) does not have the same importance as maximum shear modulus (G_{max}), it is required in modeling damping ratio when the general damping equation is used. Several researchers have studied relationships between G/G_{max} and D. Hardin and Drenvich [19] assumed that D is proportional to ($1-G/G_{max}$). Others associated D with G/G_{max} using a polynomial function [20, 21]. None of these models, however, reflect the complex relationship between plasticity index (PI) and damping. Darendeli [22] and Stokoe et al. [23] modeled the hysteretic damping assuming Masing's behavior [24] and an adjusting function to fit the Masing's damping to the experimental data, and then added a D_{min} term to obtain the total damping. The general damping equation adopted for their study has the following form:

$$D=f(G/\ Gmax)+\ Dmin \qquad (1)$$

where $f(G/\ G_{max})$ is a function of normalized shear modulus.

The most important factors that influence damping ratio include shear strain amplitude, mean effective confining stress, soil type and plasticity index, frequency of loading, and number of loading cycles. The effect of plasticity index change on damping ratio is complex, however the EPRI [25], Stokoe et al. [26], and Vucetic et al. [27] found that values of minimum damping ratio increase with the increase in plasticity index, while values of damping ratio decrease at high shear strains with increasing plasticity index. Earlier studies on damping ratio and shear modulus for composite gravel-clay soils did not show this complex effect of PI on damping ratio [28, 29]. As explained by Stokoe et al. [30], one problem with laboratory damping measurements lies in the identification of equipment-related energy loss.

On the other hand, many experimental investigations carried out on sandy and normally consolidated clayey soils in early studies [31-35] showed G_{max} was basically related to the mean effective principal stress, σ'_m and void ratio, e expressed by the well known equation:

$$G_{max}=AF(e)(\sigma'_m)^n \qquad (2)$$

in which A is an empirical constant reflecting soil fabric formed through various stress and strain histories, n is empirically determined exponent, approximately equal to 0.5 [36, 37], and $F(e)$ is void ratio function, which is usually given by:

$$F(e) = \frac{(B-e)^2}{1+e} \qquad (3)$$

where constant B is usually taken as 2.17 for round-grained sands and 2.97 for angular-grained sands [32]. Table 1 presents a summary of the empirical formulae for normally consolidated clays.

Although composite soils with properties between cohesive

Table 1. Constants in proposed empirical equations on maximum shear modulus of normally consolidated clays

(G_{max} and σ'_m in kPa): $G_{max} = A\frac{(B-e)^2 \sigma'^{0.5}_m}{1+e}$

Reference	A	B	Material
Hardin and Black [33]	3300	2.97	Clays
Hardin and Drnevich [19]	3230	2.97	Clays
Marcuson and Wahls [46]	4500	2.97	Kaolinite
	450	4.4	Bentonite
Zen and Umehara [47]	2000 ~ 4000	2.97	Remolded clay

and granular materials are found in nature enormously, unlike sands and clays, less effort has been dedicated toward understanding their dynamic behavior. This is mainly due to the inherent difficulties in characterizing heterogeneous media. The investigation described in this paper entails a study on the low-amplitude dynamic properties of sand-clay mixtures using resonant column tests. Low, medium and high plastic clays were mixed with different amounts of sand to explore the effect of the soil plasticity and aggregate content on the dynamic properties. The effect of initial density and confining stress were also investigated by preparing the specimens at three different densities and testing them under three different effective confining stresses. Eventually, on the basis of the 108 test results, a mathematical model for the maximum shear modulus is presented.

2. Tested materials and procedure

Materials tested

Low, medium and high plastic pure clays with nine mixtures of the clays with sand were used in this study. Fig.1 and Tables 2(a) and 2(b) present the grain-size distribution, and physical properties for the clays and sand-clay mixture. The sand used in the study was retrieved from a riverbed and composed of subrounded particles with minimum and maximum void ratios of 0.655 and 0.901 respectively, and a specific gravity of 2.66. Fig. 1 shows the grain-size distribution for the sand. As seen in Fig.1, all the sand particles are greater than 0.5 mm, and they will all remain on the 425μm sieve. Thus, the plasticity index of sand-clay mixtures will be that of the clay portion (Table 2b) when it is determined on the basis of the ASTM D4318-05 [38].
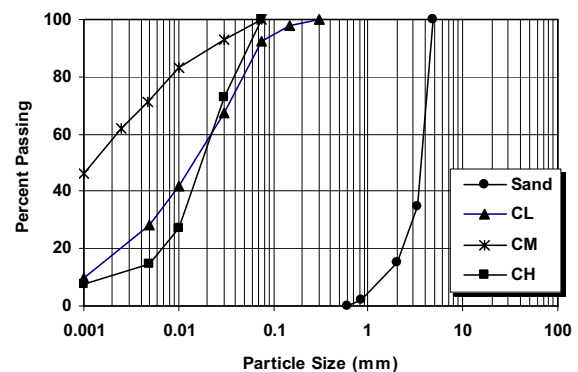

Fig. 1. Grain-size distribution for the soils used in the study; CL,CM and CH stands for low, medium and high plastic clay respectively

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Table 2(a). Physical properties of the clays

Clay Type	Plasticity Index, *PI* (%)	Specific Gravity
Low Plastic (CL)	12	2.72
Medium Plastic (CM)	25	2.70
High Plastic (CH)	50	2.71

Table 2(b). Compaction properties of the samples used in this study

Sand Content (%)	Plasticity Index, *PI* (%)	Optimum Moisture Content (%)	Maximum Dry Density (gr/cm^3)
0	12	10.4	1.81
40	12	10.6	2.00
60	12	12.6	2.06
80	12	16.8	2.05
0	25	11.3	1.58
40	25	11.5	1.89
60	25	14.4	1.98
80	25	20.0	1.97
0	50	15.5	1.42
40	50	17.6	1.71
60	50	20.5	1.81
80	50	29.4	1.97

Specimen preparation

The specimen preparation technique was chosen to model as precisely as possible the in situ condition of the core materials of embankment dams. All the specimens, typically 70 mm in diameter and 100 mm in height were prepared, with relative compactions (RC) of 80, 88 and 96% and water content of 2% wet of optimum. Relative compaction is defined as sample dry density divided by its maximum dry density obtained from standard compaction test method [39].

Appropriate amounts of clay and sand for each layer were first thoroughly mixed. Each layer was then mixed with water at least 24 hours before use and sealed. The material was poured in six layers into a cylindrical mold and compacted. To achieve a greater uniformity of specimens, a procedure similar to the undercompaction technique [40] was used. For each layer, the compactive effort was increased toward the top by increasing the number of blows per layer. Each layer was then scored after it was compacted for better bonding with the next layer.

Test procedure

The specimens were saturated with a Skempton B value in excess of 97%. To facilitate the saturation process, CO_2 was first percolated through the specimens then de-aired water was flushed into the specimens. Lastly, a back pressure of 150 kPa was incrementally applied to accelerate the saturation rate. The specimens were then isotropically consolidated under effective confining stresses of 100, 300, and 500 kPa. Figure 2 shows specimens' after-consolidation void ratios, *e* in terms of sand content. Following consolidation, torsional resonant column tests were carried out under the specifications of ASTM D 4015 [41], using a fixed-free type device.
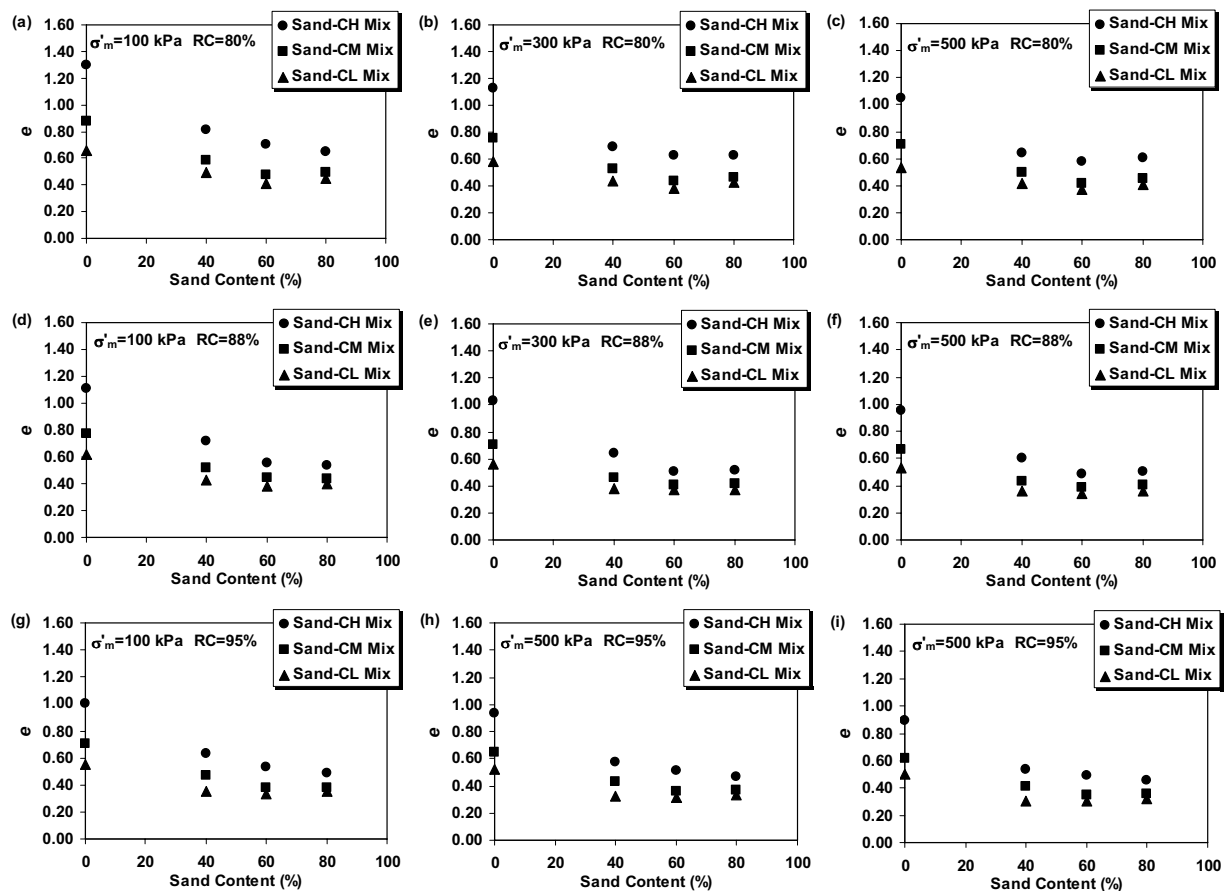

Fig. 2. Void ratio variations in sand-clay mixtures

Effect of sand content on G_{max} and D_{min}

Figure 3 presents the variation of maximum shear modulus in terms of sand content at different confining stresses (σ'_m) and relative compactions (*RC*). As may be seen, regardless of the confining stress and relative compaction value, G_{max} increases with aggregate content, until a maximum G_{max} is reached at a sand content of 60%. As the sand content continues to increase above 60%, G_{max} decreases. Vallejo and Lobo-Guerrero [42] also indicated the same trend in unsaturated mixtures of Ottawa sand with kaolinite clay. The variation of void ratio against sand content (Fig. 2) can reasonably justify the behavior shown in Fig.3. As shown in Fig.2, void ratio decreases with sand content until reaching its minimum at 60%. Beyond 60% sand content, where sandy grains prevails the soil skeleton, void ratio generally increases with sand content. Prakasha and Chandrasekaran [43] also showed that a minimum in void ratio is achieved at a sand content of 70% in marine sand-clay mixtures.

Figure 4 presents the variation of minimum damping ratio in terms of sand content at different confining stresses and relative compactions. As may be seen, regardless of the confining stress and relative compaction value, minimum damping ratio generally increases with sand content.

Effect of confining pressure on G_{max} and D_{min}

Fig. 5 depicts the variation of maximum shear modulus against confining stress (σ'_m). As seen, it appears that the rate of increase in G_{max} with confining stress (that is reflected by, exponent n in Eq. (2)) is non-linear and identical for all the mixtures. Figure 6 presents the variation of $(G_{max}/P_a)(\sigma'_m/P_a)^{0.5}$ in terms of aggregate content; where P_a is atmospheric pressure and exponent n in Eq.(2) is taken as 0.5. As seen, regardless of the density and plastic properties of the mixture, G_{max} can be successfully related to σ'_m by taking n=0.5. The effect of confining stress on minimum damping ratio is shown in Fig. 7. As can be seen, D_{min} decreases with confining stress, which is also in accordance with Zhang et al. [44] investigations on the effect of confining stress on minimum damping ratio.

Effect of clay plasticity and void ratio on G_{max} and D_{min}

The sand was mixed with low, medium and high plastic clays to investigate the effect of clay plasticity on the low-amplitude dynamic properties. The test results previously presented in Figs. 3 and 4 can be used to explore the effect of clay plasticity on maximum shear modulus and minimum damping ratio respectively. As shown in Fig. 3, for the mixtures with an identical relative compaction and confining stress, G_{max} decreases with clay plasticity. The decrease in G_{max} with clay plasticity can be justified in the light of void ratio increase with clay plasticity (Fig. 2). It is quiet evident that G_{max} is a function of void ratio (*e*), and decreases as e increases. Figure 8 presents the variation of G_{max} in terms of *e* in different mixtures used in this study. The figure clearly shows that maximum shear modulus is not a function of clay plasticity,
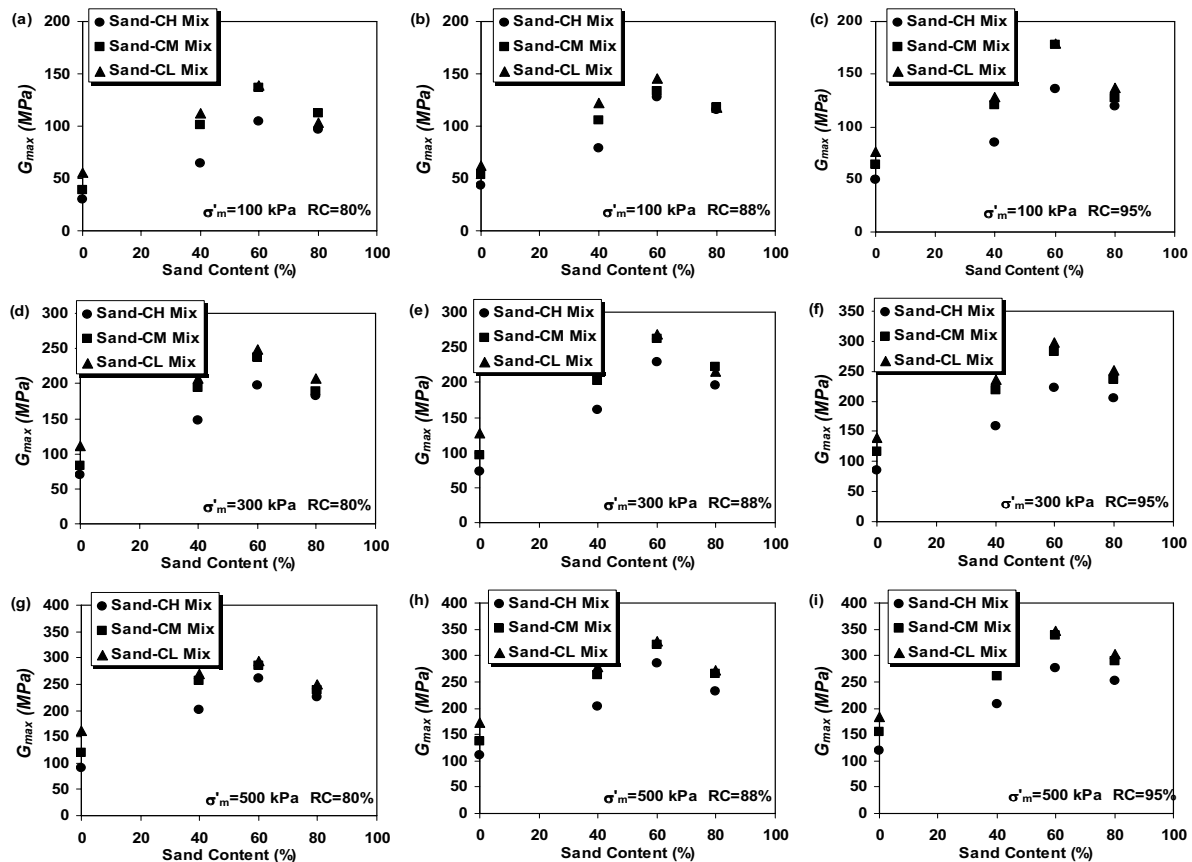

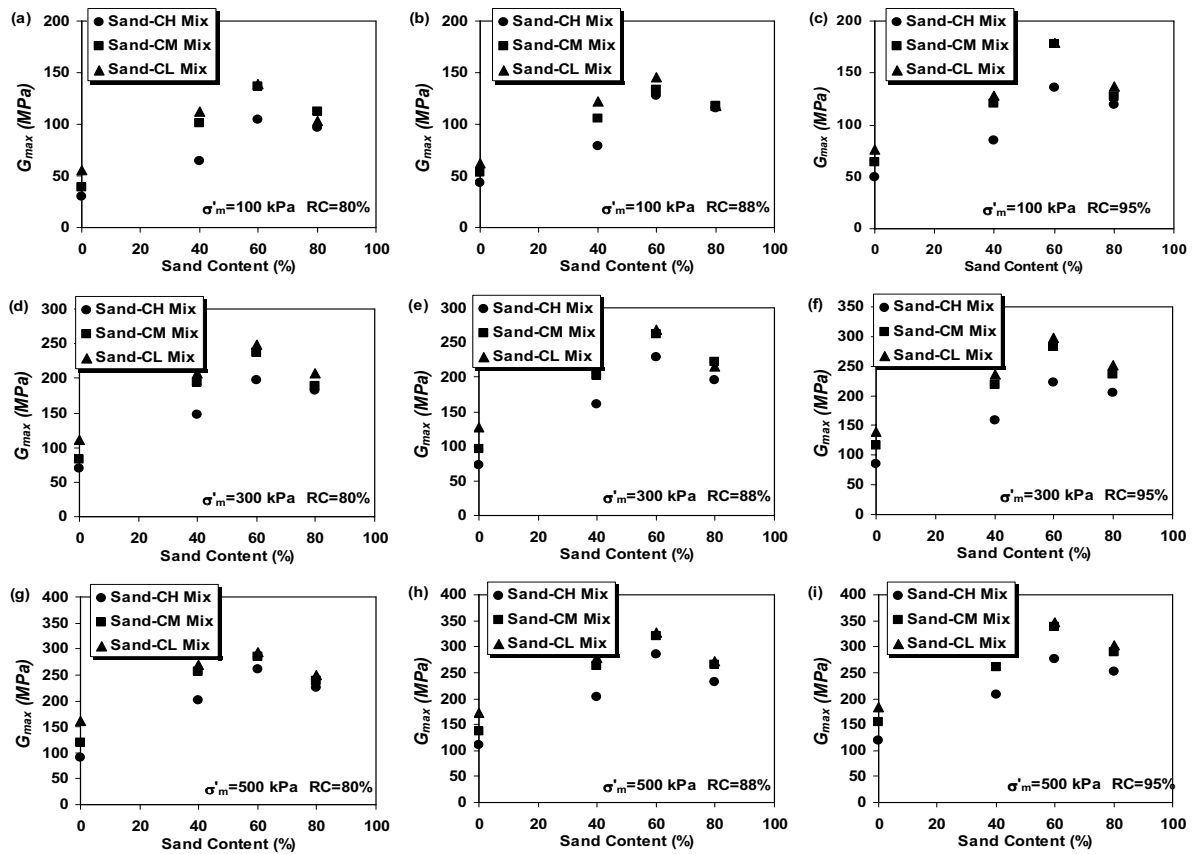**Fig. 3.** Effect of aggregate content on G_{max}
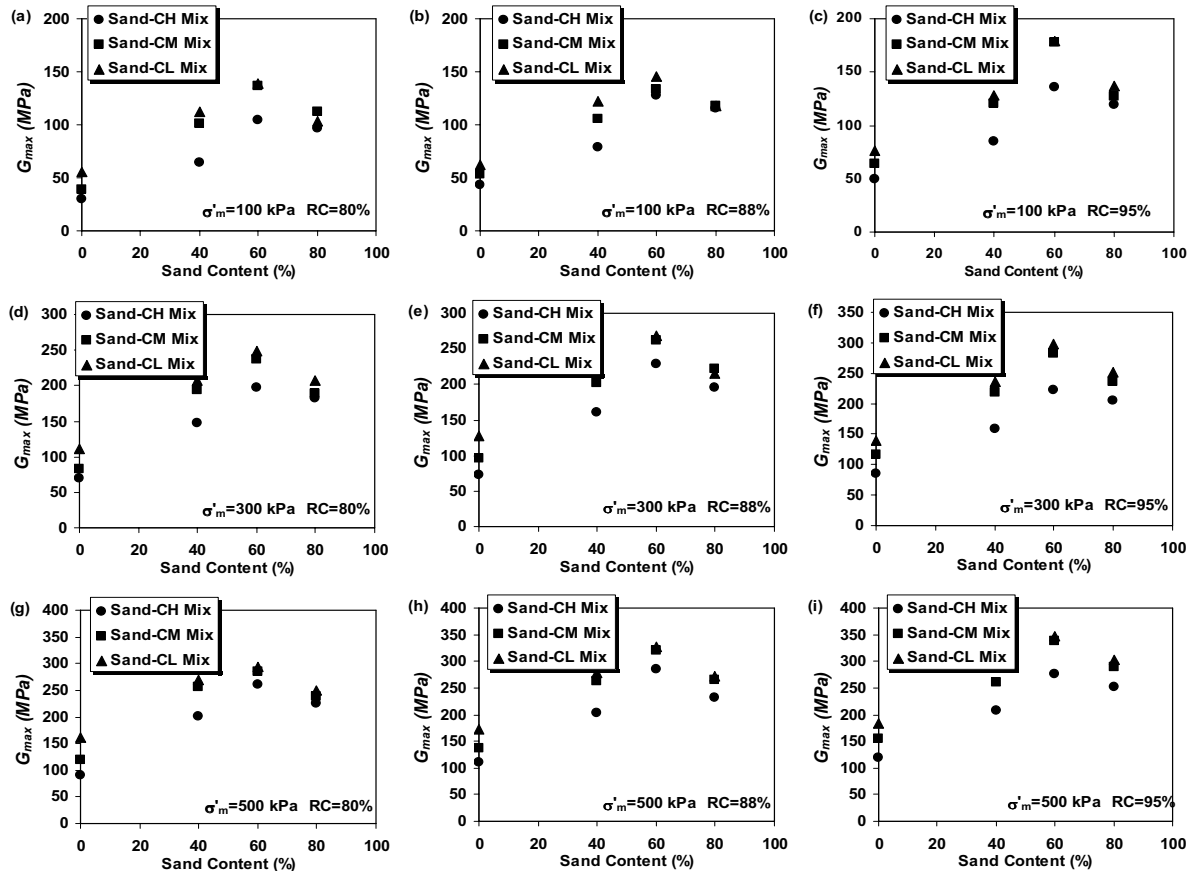
Fig. 4. Effect of aggregate content on G_{max}
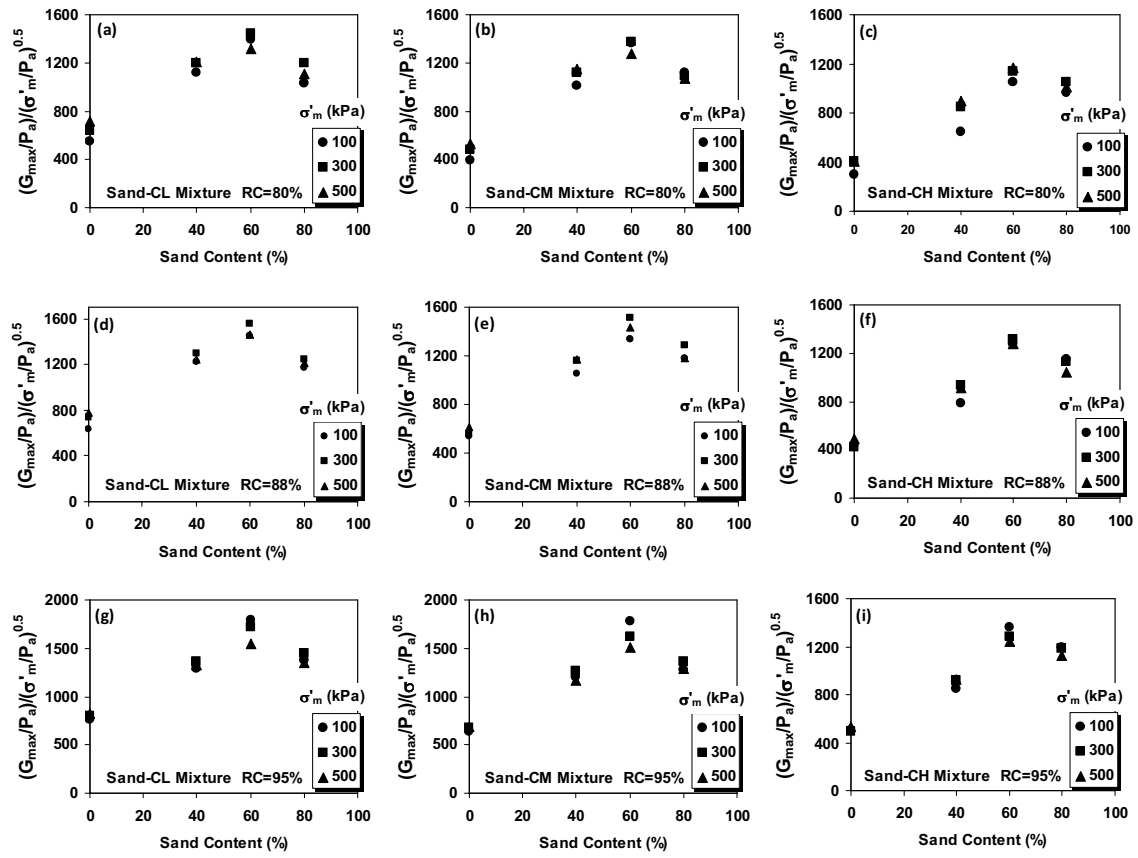

Fig. 5. Effect of confining stress on G_{max}

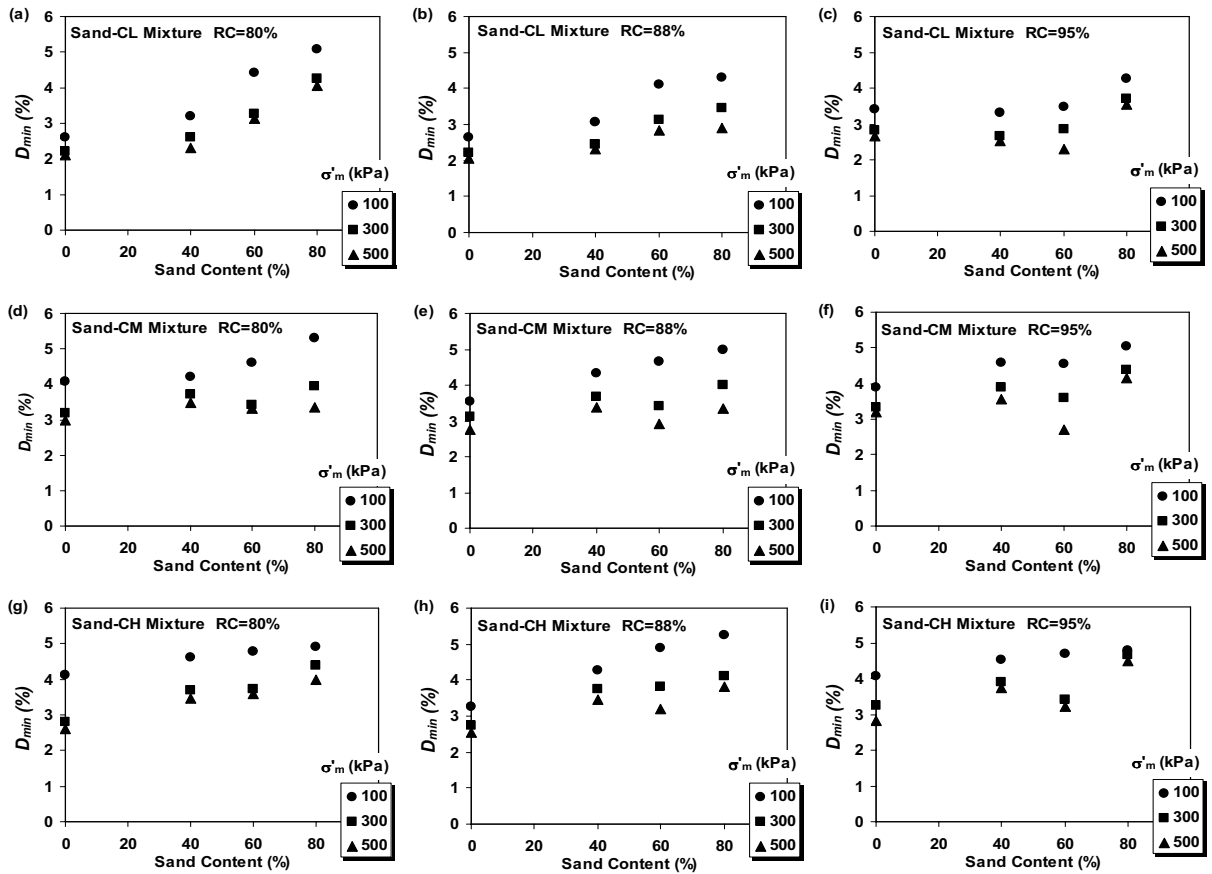Fig. 6. Normalization of Gmax with respect to confining stress (G_{max} and σ'_m in kPa)

Fig. 7. EEffect of confining stress on D_{min}

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but is a function of void ratio. Since void ratio increases with clay plasticity, mixtures containing higher plastic clays show lower maximum shear modulus.

Minimum damping ratio also increases generally with clay plasticity (Fig. 4). This is in contradict with the Vucetic and Dobry [29] curves that shows material damping decreases with soil plasticity. However, minimum damping ratio measured in resonant column apparatus is called internal damping [45], and is different from the material damping measured in cyclic triaxial or simple shear apparatus. The internal damping occurs when the energy is lost in developing friction between soil particles during stress reversals. Figure 9 can also be used to verify the effect of void ratio on minimum damping ratio. As can be seen, minimum damping ratio is almost independent of void ratio.

A mathematical model for G_{max}

It was shown that maximum shear modulus is a function of sand content, confining pressure and void ratio. Hence, to characterize the low-amplitude dynamic properties of aggregate-clay mixtures, it is necessary to find an appropriate mathematical model in the form of Eq. (2). Herein, a regression analysis based on the least square technique is used
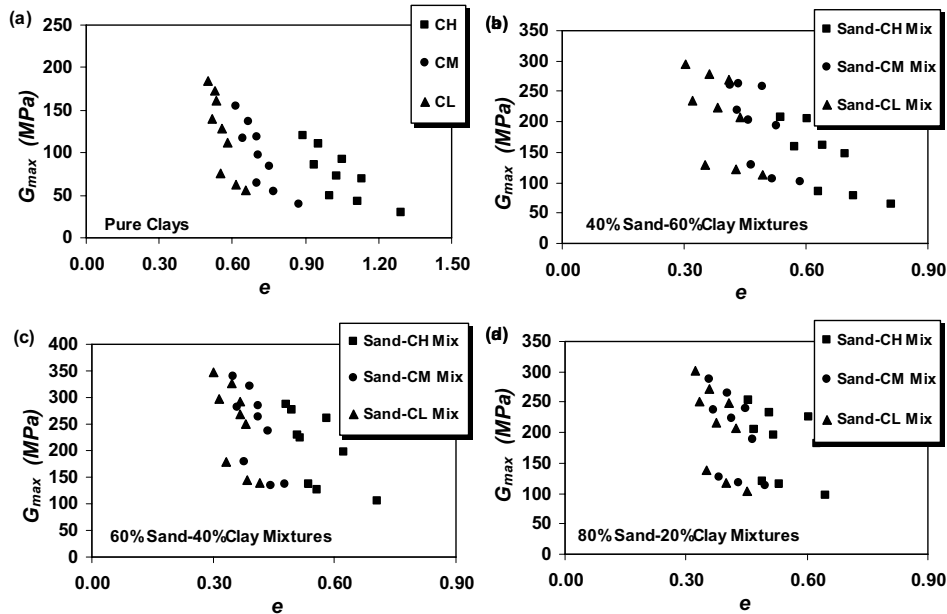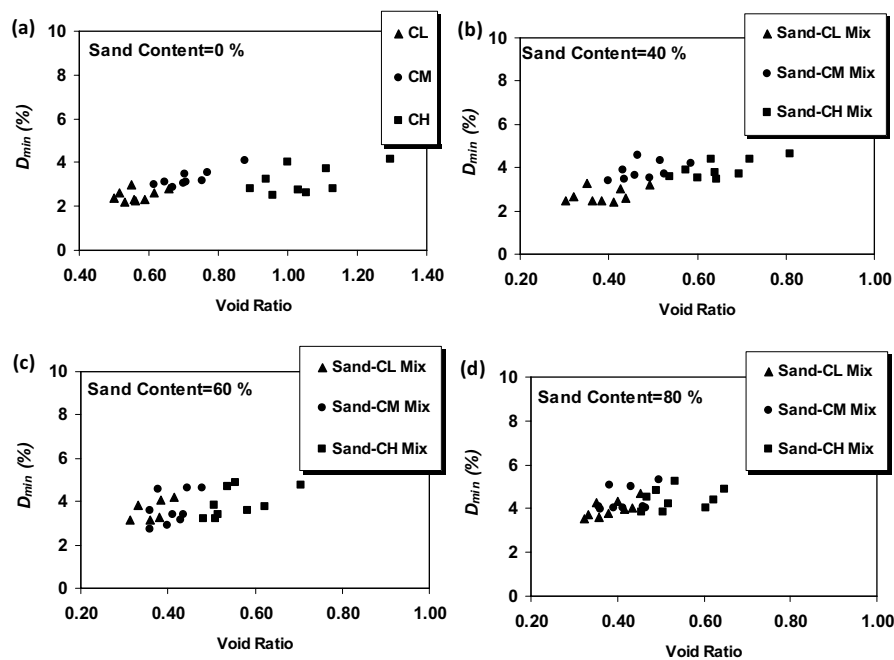

Fig. 8. Effect of void ratio on G_{max}

Fig. 9. Effect of void ratio on D_{min}

to find the values of constants A and B in Eqs.(2) and (3), assuming $n=0.5$. The analyses are carried out by varying B for each mixture until achieving a coefficient of determination, R^2 more than 95%. The criterion will be satisfied if a value of 2.95 is assumed for B. Figure 10 depicts the variation of $(G_{max}/P_a)(\sigma'_m/P_a)^{0.5}$ in terms of $F(e)$ for different mixtures, where B is equal to 2.95. As seen, a line can be successfully fitted to the data. The slope of this line is $A/P_a^{0.5}$, where P_a is atmospheric pressure, and is equal to 100 kPa. It is interesting to note that the value of B obtained for sand-clay mixtures is very close to 2.97, that is proposed in other studies for clays (Table 1). Table 3 shows the values of A and R^2 for each mixture when $F(e)$ is equal to $(2.95-e)^2/(1+e)$. As seen, value of constant A varies from 2032 to 3066 which falls well into the limits proposed in other investigations for clays (Table 1). Figure 11 shows the variation of A against sand content. As seen, constant A linearly increases with aggregate content until a sand content of 60%. As the sand content continues to increase above 60%, A decreases. Thus, A can be described as a function of sand content (SC) by the following linear equations (Fig.12):

$$A = 1700SC + 2000 \qquad SC \leq 0.60 \qquad (4)$$

$$A = -2000SC + 4300 \qquad 0.60 < SC \leq 0.80 \qquad (5)$$

The variation of G_{max} in terms of e along with the proposed model for each mixture is shown in Fig.13. As seen, the model successfully predicts maximum shear modulus.

Conclusions

An experimental study was performed on the compacted pure clays and mixtures of sand-clay to investigate the effect of sand content, confining stress, dry density and plasticity of the clayey part on the low-amplitude dynamic deformation properties using resonant column tests. A mathmatical model was also developed for maximum shear modulus, G_{max} of sand-clay mixtures. The following conclusions may be drawn based on this experimental study:

1. maximum shear modulus increases with aggregate content, until a maximum Gmax is reached at a sand content of 60%. As the sand content continues to increase above 60%, G_{max} decreases. In addition, maximum shear modulus increases nonlinearly with confining stress, so that it has a good correlation with square root of confining stress. maximum shear modulus is not a function of clay plasticity, but is a function of void ratio. Since void ratio increases with clay plasticity, mixtures containing higher plastic clays show lower G_{max};

2. minimum damping ratio increases with sand content and soil plasticity, however it decreases with confining stress. In
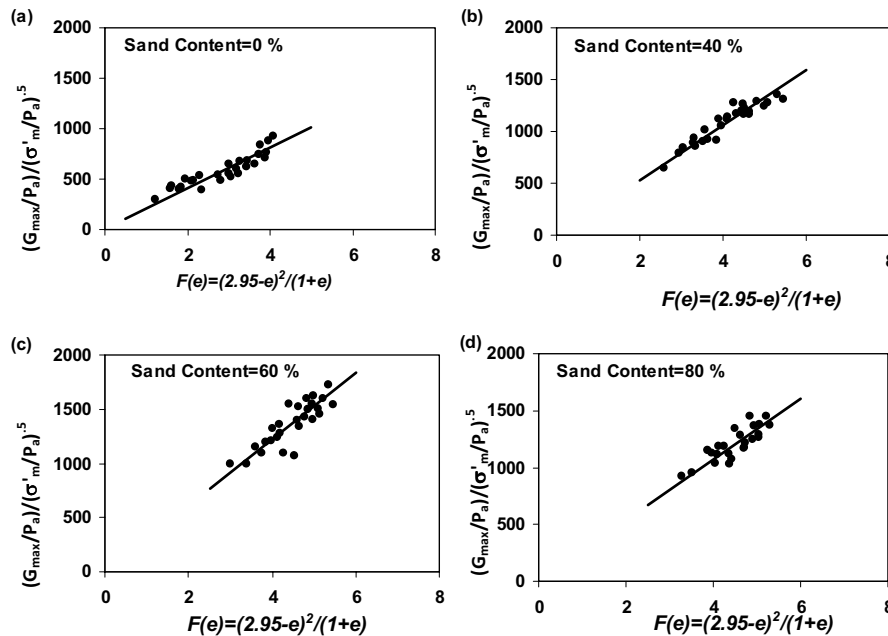

Fig. 10. Void ratio function for sand-clay mixtures

Table 3. Values of constant A in Eq. (2) and R² for the mixtures; (G_{max} and σ'_m are in kPa and B=2.95)

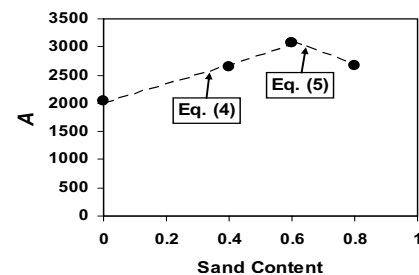| Aggregate Content (%)	A	R^2
0	2032	0.95
40	2658	0.98
60	3066	0.95
80	2671	0.98

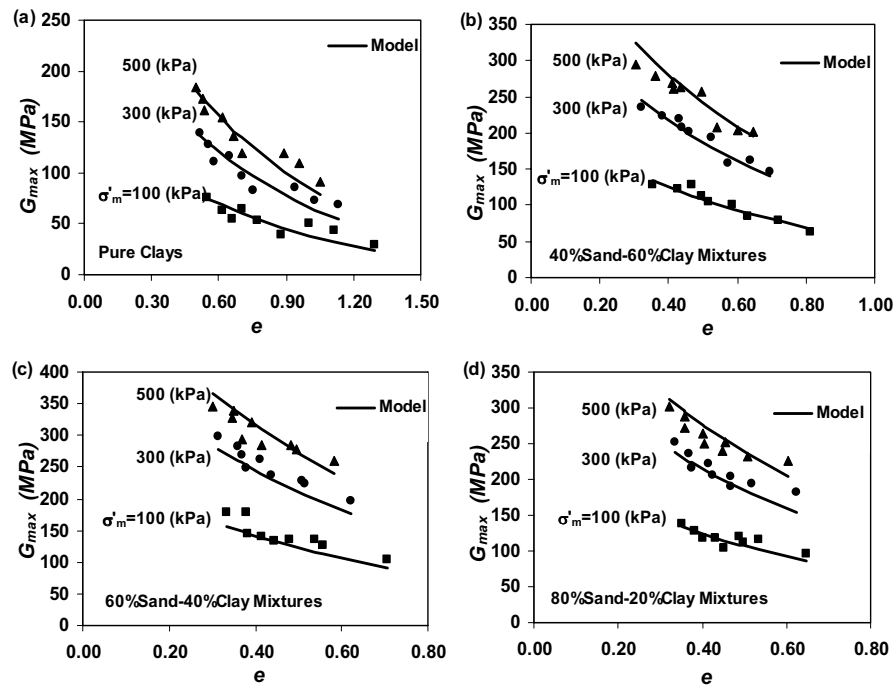**Fig. 11.** Constant A as a function of sand content

Fig. 12. Void ratio function for sand-clay mixtures

addition, Dmin is not affected by the specimen void ratio;

3. A mathematical model was developed for maximum shear modulus of the mixtures examined in this study. The model is very similar to Hardin and Drnevich (1972) model for normally consolidated clays and predicts G_{max} as

$$A \frac{(2.95 - e)^2 \sigma'^{0.5}_m}{1 + e}$$

where A is a function of sand content, e is void ratio and σ'_m is mean effective principal stress.

Notation

CH = High plastic clay
CL = Low plastic clay
CM = Medium plastic clay
D = Damping ratio
D_{min} = Minimum damping ratio
$F(e)$ = Void ratio function
G = Shear modulus
G_{max} = Maximum shear modulus
e = Void ratio
P_a = Atmospheric pressure
PI = Plasticity index
R^2 = Coefficient of determination
RC = Relative compaction
SC = Sand content
σ'_m = Confining pressure

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