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Optimal plastic design of frames using evolutionary structural optimization

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Abstract

An evolutionary structural optimization (ESO) method is used for plastic design of frames. Based on safe theorems some criteria are derived and made an effort to satisfy them during the optimization process. In this regard, equilibrium is checked and yield condition is gradually satisfied during the optimization process. In this method, the amount of used material and the stiffness for each element are improved, simultaneously, to impose upper bound of moment in the element. Frame analysis and optimization algorithm are implemented as PLADOF (PLAstic Design of Frames) computer code. Four examples are presented to illustrate the performance of the algorithm.

Keywords: Optimization, Plastic design, Evolutionary methods, Safe theorem, Plastic moments.

1. Introduction

Equilibrium, compatibility and constitutive relationship are the general tools which should be satisfied in elastic analysis of structures. In plastic analysis and design, equilibrium, mechanism and yield are three conditions which should be fulfilled in order to obtain the unique solution. Since the design spaces in plastic problems are complicated, thus the lower and upper bounds of the solution are often explored. By satisfying the equilibrium and selecting an arbitrary moment distribution in which no moment exceeds the plastic moment of the members, i.e. satisfying the yield condition, the safe or lower bound approach is formulated. The maximum load obtained by this method represents the plastic limit load. On the other hand, if a mechanism is assumed for a structure and the equilibrium is satisfied, then the unsafe or upper bound approach is created. The best value of the plastic limit load is the lowest value obtained from all mechanisms considered. It should be noted that the mechanism and yield conditions might be violated in lower and upper bound approaches, respectively. Hence, plastic analysis and design can be interpreted as an optimization problem [1,2].

Plastic analysis and design of frames was cast in the form of linear programming by Charnes and Greenberg [3], as early as 1951. Further progress in the field is due to Heyman [4], Horne [5], Baker and Heyman [6], Jennings [7], Kirsch [8], Theirauf [9], Kaveh [10], Kaveh and Khanlari [11] and Kaveh and Mokhtarzadeh [23] among many others.

In recent years finite element analysis (FEA) has become a widely used tool for practicing engineers of many disciplines. Structural optimization, however, has achieved far less popularity in practice despite the extraordinary progress of the optimization theory and associated algorithms over the past three decades. This situation is caused to a large extent by the mathematical complexities of the existing optimization methods.

Since 1992 researchers have tried to bridge the gap between FEA and structural optimization by developing a very simple approach to optimal structural design. It is based on the concept of slowly removing inefficient materials from a structure so that the residual structure evolves towards the optimum. This is named evolutionary structural optimization (ESO).

The ESO method proves to be capable of solving size, shape and topology structural optimization for static, dynamic, stability and heat transfer problems or combinations of these [12,13]. The ESO method appeals to practicing engineers and architects particularly because of its simplicity and effectiveness.

In order to optimum design of frames several optimization methods such as heuristic algorithms [15-20], optimality criteria [21] and graph theoretical [22-23] methods are used. In this paper, the ESO method is utilized for plastic design of two dimensional frames and some criteria are derived in order to form a simple structural evolutionary optimization problem. The total weight of the structure is minimized and by making

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use of safe theorem the stiffness of each member is adjusted according to limited corresponding moments. In other words, the weight of the structure is minimized while the equilibrium and yield condition are satisfied. Four examples are considered to verify and illustrate the performance of the method.

2. Optimization problem

The aim of the optimization problem is to minimize the weight of structure while, based on safe theorem, equilibrium and yield condition to be fulfilled. Therefore, the optimization problem can be written as follows

$$\begin{aligned} \min_{\alpha^e, \boldsymbol{\beta}^e} \quad & Weight \\ s.t. \quad & Equilibrium \\ & \left|M^e(\boldsymbol{\beta}^e)\right| \le M_p^e(\alpha^e) \\ & 0 \le \alpha^e \le 1 \\ & 0 \le \boldsymbol{\beta}^e \le 1 \end{aligned} \tag{1}$$

where α^e are β^e design variables of optimization problem, $M^e(\beta^e)$ are existed moment at the ends of element e which varies depending on the stiffness of the element, $M_p^e(\alpha^e)$ is the plastic moment of member e in each iteration of optimization which depends on α^e. In this method, α^e can be interpreted as an index for minimizing the weight of element and β^e are indices for satisfying the yield condition at the both ends of the element e.

Let $\overset{u}{M}{}_p^e$ be the upper bound of plastic moment for element e which is obliged by user because of construction constraints. Then, $M_p^e(\alpha)$ can be expressed as

$$M_p^e(\alpha) \doteq \alpha \overset{u}{M}{}_p^e \tag{2}$$

Since $0 \leqq \alpha e \leqq 1$, therefore, $0 \le M_p^e(\alpha) \le \overset{u}{M}{}_p^e$ It is noticed that if $\alpha e=0$, it means that the element e can be removed from the structure layout whereas $\alpha e=1$ means that the biggest available section (plastic moment) should be used for element e. In this situation ($\alpha e=1$) obtained moment from elastic analysis can be greater than plastic moment i.e. the yield condition is violated. In order to satisfy the yield condition, β is introduced which can be written as below for one dimensional linear finite element

$$\boldsymbol{\beta} = \begin{Bmatrix} \beta_1 \\ \beta_2 \end{Bmatrix} \tag{3}$$

where superscripts 1 and 2 refer to the start and the end point of the element e as shown in Figure 1.

Force-displacement relationship for each finite element can be written as

$$\mathbf{f}^e = \mathbf{k}^e \mathbf{u}^e \tag{4}$$

Equation (4) should be modified if one end of the member has a hinge that causes the corresponding force to be equal to zero [14]. The n^{th} component of f^e can be written as follows

$$f_n^e = \sum_{j=1}^{6} k_{nj}^e u_j^e \tag{5}$$

If $f_n^e = 0$ then corresponding displacement component can

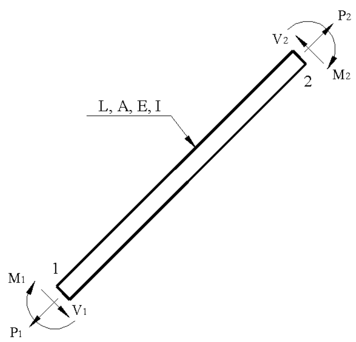


Fig. 1. Typical member of a frame

be written as [14]

$$u_n^e = \sum_{j=1}^{n-1} \frac{k_{nj}^e}{k_{nn}^e} u_j + \sum_{j=n+1}^{6} \frac{k_{nj}^e}{k_{nn}^e} u_j \tag{6}$$

By substituting (6) into the other five equilibrium equations, the unknown u_n can be eliminated and the corresponding row and column are set to zero. Therefore, (4) should be replaced by

$$\bar{\mathbf{f}}^e = \bar{\mathbf{k}}^e \mathbf{u}^e \tag{7}$$

The new stiffness terms are as follows [14]:

$$\bar{k}_{ij}^e = k_{ij}^e - k_{in}^e \frac{k_{nj}^e}{k_{nn}^e} \tag{8}$$

This procedure is applied to release the n^{th} component of f^e matrix. By introducing the parameter β, the related component of f^e can be evaluated according to the values of .

$$\bar{k}_{ij}^e(\beta^e) \doteq k_{ij}^e - \beta^e \left(k_{in}^e \frac{k_{nj}^e}{k_{nn}^e}\right) \tag{9}$$

where varies from zero to one, i.e. $0 \leqq \beta^e \leqq 1$. If β^e is equal to one, then the related degree of freedom is released, and if it is equal to zero, then $\bar{k}_{ij}^e = k_{ij}^e$. According to (5) moments at the start and the end of members can be written as follows

$$M_1^e(\beta_1) = \sum_{k=1}^{6} \bar{k}_{31k}(\beta_1) u_k \tag{10}$$

$$M_2^e(\beta_2) = \sum_{k=1}^{6} \bar{k}_{6k}(\beta_2) u_k \tag{11}$$

3. Evolutionary structural optimization algorithm

The evolutionary structural optimization (ESO) method is based on the simple concept of gradually removing underutilized material from a structure so that the resulting form evolves towards an optimum [12, 13]. Here, the material of a member is reduced when its moments are little than plastic moment and will be increased when those are bigger. On the other hand, since a member can not be considered bigger than a maximum considered section, therefore, the flexural stiffness of member should be modified according to (11) so that the corresponding moments obtained to be at least the same as plastic moments. The procedure should be repeated until all moments are little than plastic moments in the frame. Based on the above procedure the optimization algorithm can be written as follows

1. Optimization algorithm should be started by the initial following design variables

$$\alpha^e = 1, \quad \beta_1^e = 0 \quad and \quad \beta_2^e = 0. \tag{12}$$

In other words the first iteration is commenced by the upper bound of plastic moments and all connections are rigid.

2. Frame elastic analysis using above design variables is done and internal forces are derived.

3. In order to find a criterion, the variable is defined as below

$$sign = \left|M_{\max}^e(\boldsymbol{\beta})\right| - M_p^e(\alpha) \tag{13}$$

where $M_{\max}^e(\boldsymbol{\beta})$ is the maximum moment along member *e*, obtained from elastic analysis.

4. If *sign* is a positive number then should be increased and if *sign<0* then should be decreased in order to move toward optimum. If α to be greater than 1 it means that the member can not sustain the obtained internal moment. Therefore, β should be varied. Since α can not be greater than one, this value is replaced by 1 at this stage.

5. Following the true condition α=1, in this stage β is varied. In order to achieve this, ζ is evaluated as below

$$\zeta_{1,2} = \left|M^e(\beta_{1,2})\right| / M_p^e(\alpha) \tag{14}$$

For α=1 and using Eq. (2), $M_p^e(\alpha)$ can be obtained and replaced by $\overset{u}{M}{}_p^e$ Therefore

$$\zeta_{1,2} = \left|M^e(\beta_{1,2})\right| / \overset{u}{M}{}_p^e \tag{15}$$

If ζ_i is little than one then β_i should be decreased otherwise β_i is increased. It is noted that β_i can not be grater than one or little than zero.

6. The structure is reanalyzed using new design variables and the internal forces are derived.

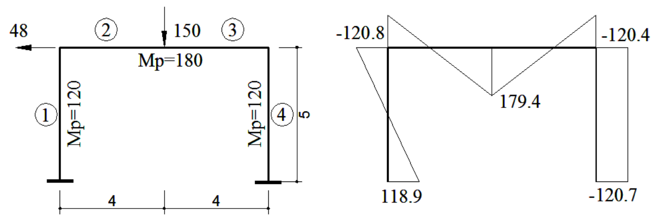
7. The evaluated internal moments are compared to $M_p^e(\alpha)$ to be smaller than the permissible tolerance imposed by user, thus the process is terminated. Otherwise, the algorithm should be repeated from Stage 3.

4. Numerical examples

In order to illustrate the performance of the algorithm, the following four examples are presented. The move limits are considered to be 0.01 for α, and 0.005 for β in order to approach to optimum.

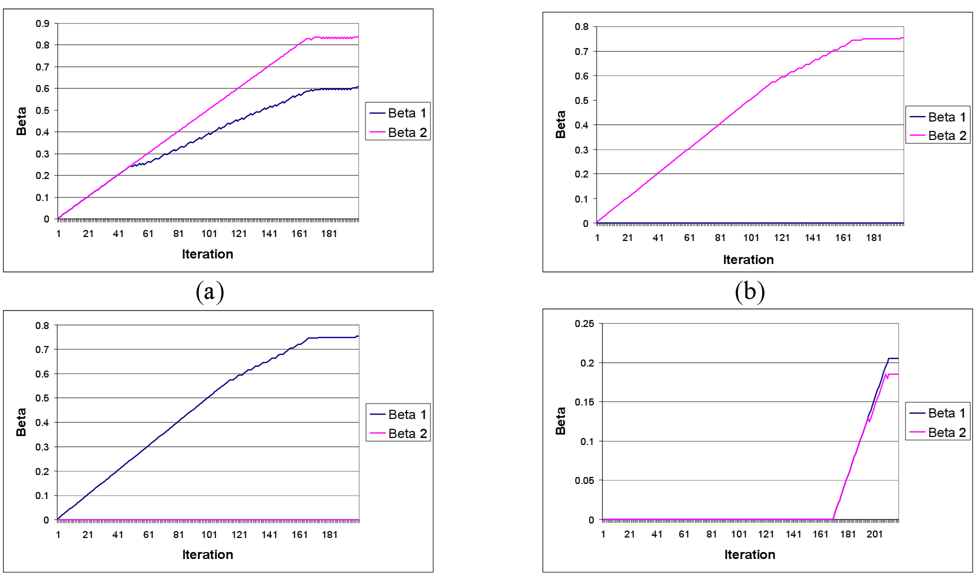
Example 1. In order to verify the method a portal frame from reference [24] is considered as shown in Figure 2(a). After optimization process the moment distribution diagram is obtained as illustrated in Figure 2(b). It is shown that the maximum error occurred at the top left joint of the frame, which has a difference of 0.7% compared to its exact value of maximum plastic moment. This amount can be considered as negligible.

The variation of design variables β_1 and β_2 for each member during the optimization algorithm has been shown in Figure 3. From Figure 3(d) it is observed that last hinges occurred in element No. 4 after 170 iterations.



(a) (b)

Fig. 2. Portal frame of Example 1 (a) geometry, boundary conditions and upper bounds of plastic moments (b) diagram of the distribution of moments.



(a) (b) (c) (d)

Fig. 3. Variation of design variables and during optimization process for each finite element (a) element 1 (b) element 2 (c) element 3 (d) element 4.

As the second part of this example, it is assumed that there is no constraint for beam section. Thus, the upper bound of plastic moment for beam can be considered to be a large value such as $\ddot{M}_p = 1800$ kN.m. Therefore, by considering the multiplication of length and plastic moment of a member as the weight of member, the weight of the frame is assumed 15450 kN.m^2 for the first optimization step. After the optimization process the distribution of moments is illustrated in Figure 4(a). From the results, it is observed that the weight is reduced to 2628 kN.m^2. It is noted that in reference [24] the weight of frame is obtained equal to 2640 kN.m^2 by using the simplex method.

It is interesting to note that the yield condition is not violated because of using relaxed optimization space in this part. However, the results are not changed much in comparison to Figure 2(b). As shown in Figure 5(a) and 5(d) the columns weight is not changed but the weight of beams is reduced during the optimization and there is not any hinge in beams

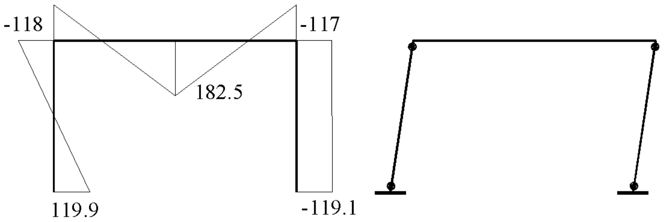


Fig. 4. (a) Moment distribution in optimized frame (b) collapse mode

according to Figures 5(b) and 5(c).

Example 2. Inspired from the example of plastic moment distribution method in Reference [2], a 1-bay and 3-story frame depicted in Figure 6(a) is considered. For the first part of the example, beams are assumed to be more relaxed. To achieve this, the upper bounds of plastic moments for beams are considered as shown in Figure 6(a). After optimization

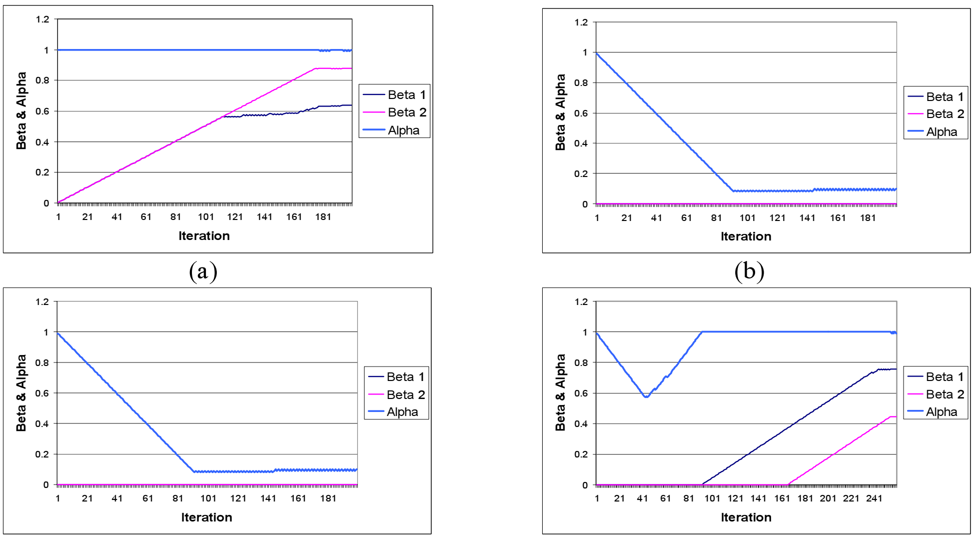


Fig. 5. Variation of design variables , β_1 and β_2 during optimization process for each finite element (a) element 1 (b) element 2 (c) element 3 (d) element 4.

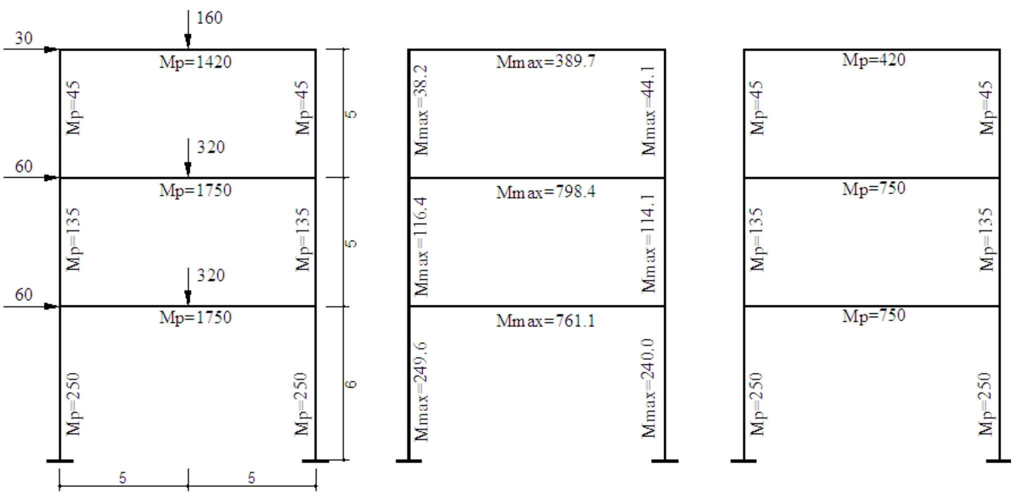


Fig. 6. 1-bay and 3-story frame (a) geometry, boundary conditions and upper bound of plastic moments (b) maximum obtained moment for each member (c) selected plastic moments in Reference [2].

process the absolute value of maximum moment for each member is obtained as depicted in Figure 6(b). Also, the minimum selected plastic moments for beams and columns in reference [2] are shown in Figure 6(c). It is observed that the optimum weight of the frame is 23994 kN.m^2 that is very close to 24000 kN.m^2 which is considered in Reference [2].

For the second part, minimum beam sections are considered and columns are assumed more relaxed, as illustrated in Figure 7(a). The results are shown in Figure 7(b). Variation of weight during the optimization process is depicted in Figure 8.

Example 3. The illustrated frame in Figure 9(a) is studied. The upper bounds of the plastic moments and loading are shown in this figure. After the optimization process, the results show that the highest error with the magnitude 1.4% occurs in the plastic moment of a beam. In this example, the weight is decreased from 108849.6 to 89366.4. The design variables for two elements i and j are varied as illustrated in Figure 10.

Example 4. A 2-bay pitched roof frame is considered. Geometry, loading and the plastic moments of sections are shown in Figure 11. The final maximum moments are specified in Figure 12. The weight of frame is decreased from 15899 kN.m^2 to 2163 kN.m^2 during the optimization process.

5. Conclusions

The proposed algorithm, based on evolutionary structural optimization, can be used as a robust tool for optimal plastic design of frames. Appropriate upper bounds of plastic moments are first chosen according to the value of applied loads. It is noted that some constrains, such as assumption of minimum beam or column sections, can be imposed. However, special care needs to be taken for choosing plastic moments. After that, the proposed optimization algorithm is used to derive the minimum weight while the yield condition is not violated. The accuracy of the results is shown via some examples from the literature.

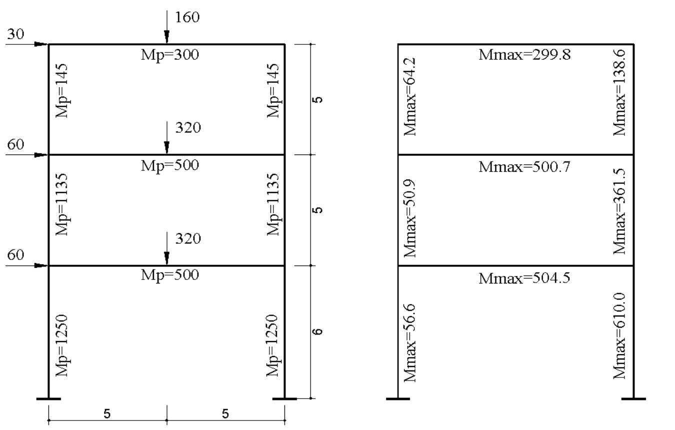


Fig. 7. 1-bay and 3-story frame (a) geometry, boundary conditions and upper bound of plastic moments (b) final design

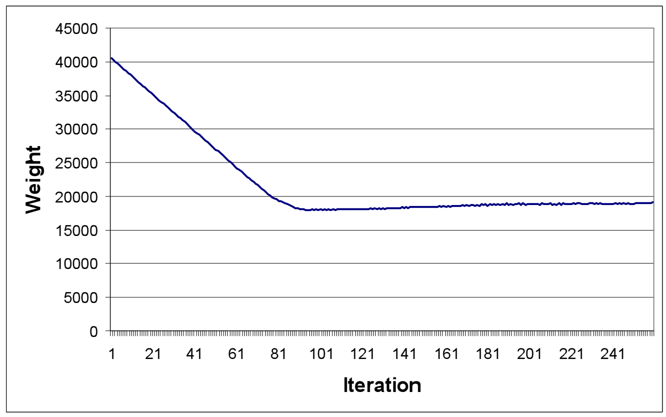


Fig. 8. Variation of weight during optimization iterations

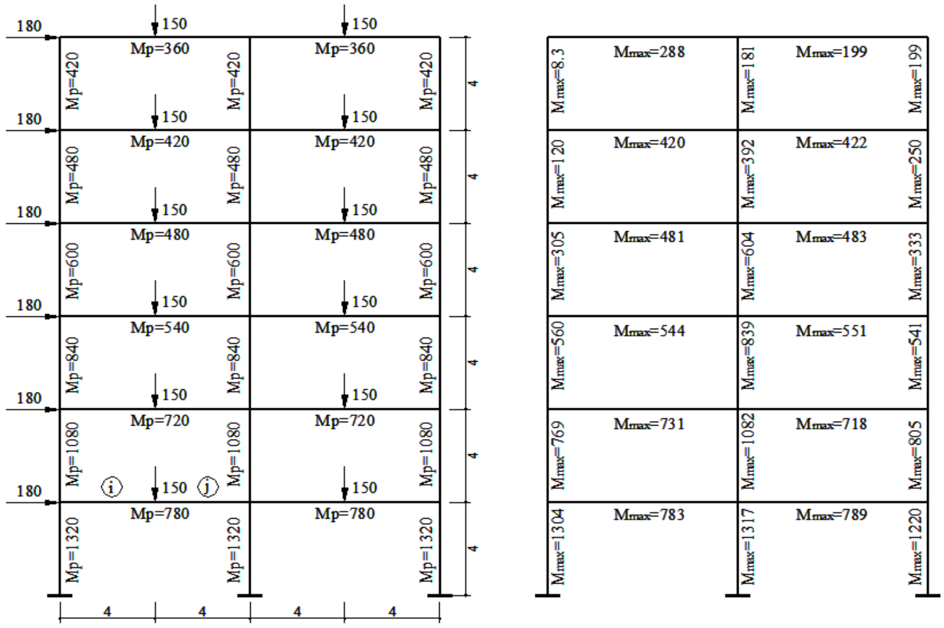
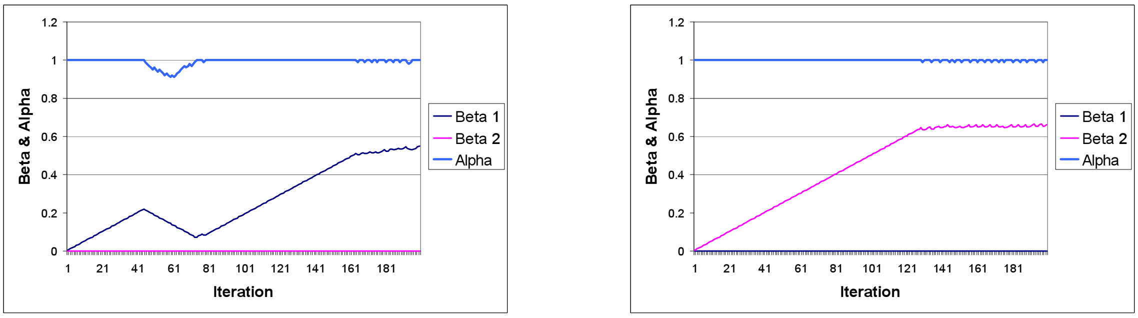
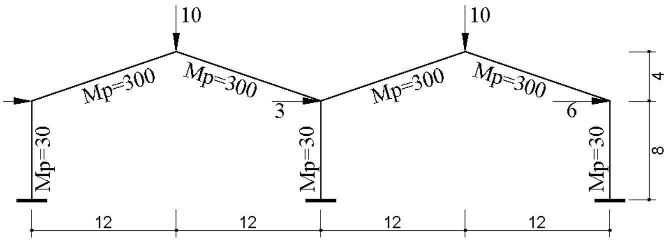


Fig. 9. 2-bay and 6-story frame (a) geometry, boundary conditions and upper bound of plastic moments (b) final design



(a) (b)

Fig. 10. Variation of design variables , and during optimization process for elements (a) i and (b) j



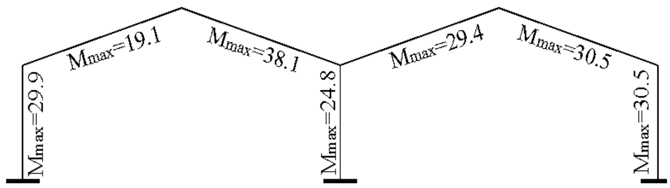


Fig. 11. A 2-bay pitched roof frame geometry, boundary conditions and upper bound of plastic moments

Fig. 12. Final maximum bending moments

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