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Technical Note

A numerical approach for one dimensional thermal consolidation of clays

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Abstract

In saturated soils, heating induces thermal expansion of both grains and the pore fluid. Lower thermal expansion coefficient of aggregates results in the increase of pore pressure and reduction of the effective stress besides subsequent volume changes due to the dissipation of pore pressure and heat transfer. Dissipation of thermally induced pore pressure with time is a coupled thermo-hydro-mechanical (THM) phenomenon, involving gradients of pore pressure and temperature, hydraulic and thermal flows within the mass of soil and changes in the mechanical properties with temperature. The objective of this paper is presentation of a numerical method to determine the effect of temperature on consolidation of clays. In this regard, the finite element code, PISA is used for one dimensional THM analysis of porous media. The analysis performed using both linear elastic and elastoplastic Cam clay models. Modified Cam clay model was applied in elastoplastic analysis. Variation of temperature, displacements and pore pressure determined with time and compared with numerical solutions of other researchers. Also it was indicated that implementation of coupled THM analysis yields better results for displacements compared to the hydro mechanical (HM) one. Application of elastoplastic constitutive model instead of linear elastic one indicated that preconsolidation pressure has an important effect on results of analysis.

Keywords: *THM analysis, Thermal consolidation, Coupled phenomenon, PISA, Finite element.*

1. Introduction

Earliest recognition of the multiphase nature of geomaterials is generally attributed to Terzaghi [1]. Terzaghi postulated that when a low permeability soil such as clay is subjected to an external loading, the load is initially carried by pore fluid rather than the solid skeleton. Consequently, the pore fluid pressure immediately increases after application of external loading. As time progresses, flow of pore fluid takes place from regions of high pore pressures to regions of lower pressure. This pore fluid redistribution results in a gradual pore pressure dissipation accompanied by a gradual transfer of external loading to the solid skeleton. Gradual increase in stresses within solid skeleton resulting from load transfer leads to the change in geometrical configuration of solid assemblage and reduction in pores volume which manifests in form of soil consolidation.

Original developments of Terzaghi followed later by Biot's theory of isothermal consolidation of elastic porous media [2]. Literature on the theory of isothermal consolidation is quite extensive. In view of the interest in application of theory of isothermal consolidation to practical problems, attention has been focused mainly on numerical methods such as finite element method and boundary integral equation schemes. These numerical methods allow the development of approximate solutions even for complex equations, boundary conditions and material behavior.

Finite element method has been the most widely used in thermo mechanical problems. Earliest application of finite element method for study of isothermal consolidation problems is due to Sandhu and Wilson [3]. This study was followed by works of many other investigators including Christian and Boehmer [4], Ghaboussi and Wilson [5].

Soils surrounding nuclear waste disposal reservoirs or buried high-voltage cables are exposed to an elevated temperature during long periods of time and suffer changes of their mechanical and hydraulic properties. The study of thermal effects on engineering characteristics of soils dates back to the late 30s. Booker and Savvidou [6] developed an analytical solution for the problem of a point heat source buried deep in a saturated soil. Integrating the same solution, they developed a solution for a spherical heat source [7]. Aboustit et al. [8] also investigated the consolidation phenomenon around the nuclear waste reservoirs.

Non-isothermal consolidation of deformable porous media is the basis of more recent coupled THM models. Gosavi [9] developed *T2STR* code for thermo-hydro-mechanical

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problems. Sanavia et al. [10] presented a new finite element formulation for THM analysis of elastoplastic multiphase materials based on mechanics of porous media. Other researchers have proposed numerical methods for solution of coupled THM processes in porous media [11, 12, 13, 14, 15]
The objective of present paper is proposing a numerical model for THM analysis of water saturated non-isothermal porous material. Porous media is treated as a two-phase system composed of the solid skeleton (superscripted by "s") besides open voids completely filled by water (superscripted by "w"). The analysis is performed using both linear elastic and elastoplastic Cam-clay constitutive models to investigate the effect of applied constitutive relations on the answers. Solutions for linear elastic model have been compared with available numerical results of other researchers [16, 17].

2. Governing Equations

Mathematical model and its implementation in finite element code considers the soil as a non-isothermal two-phase media in which water fill voids considering the following coupled equations.

2.1. Balance equation

Assuming porous media to be constituted of incompressible soil and water components, balance equation can be derived. Linear momentum balance equation in terms of Cauchy's effective stress σ' for porous media assumes the following form:

$$div(\sigma' - P^w 1) + \rho g = 0 \quad (1)$$

where, P^w is pore pressure and p is density of porous media proposed by Eq. (2):

$$p = (1-n)p^s + np^w \quad (2)$$

In which, n is porosity and ρ^s and ρ^w are densities of solids and water respectively. Also, g is gravity cceleration vector and l is the second order identity tensor.

2.2. Mass conservation equation

In mass conservation equation, water flux has been described using the Darcy's law and has the following form:

$$\rho^w div V^s - div\left(\rho^w \frac{K}{\mu^w}[grad(P^w) - \rho^w g]\right) - \rho^w \beta^{sw}\frac{\partial T}{\partial t} = 0 \quad (3)$$

where V^s is solids velocity, K is the intrinsic permeability and μ^w is water absolute viscosity. β^{sw} is the cubic thermal expansion coefficient of medium which can be obtained according to Eq. (4):

$$\beta^{sw} = (1-n)\beta^s + n\beta^w \quad (4)$$

where β^s and β^w are solids and water cubic thermal expansion coefficients respectively.

2.3. Energy balance equation

Neglecting terms related to the mechanical work induced by density variations due to temperature changes and considering heat transfer through conduction and convection, energy balance equation of porous media can be obtained in form of Eq. (5):

$$(\rho c_p)_{eff}\frac{\partial T}{\partial t} - \left(\rho^w c_p^w \frac{K}{\mu^w}\left[grad(P^w) - \rho^w g\right]\right)grad(T) - div\left[\chi_{eff}\, grad(T)\right] = 0 \quad (5)$$

where $(\rho c)_{eff}$ is the effective thermal capacity of porous media proposed by Eq. (6) and χ_{eff} is the effective thermal conductivity of porous media.

$$(\rho c_p)_{eff} = (1-n)\rho^s c_p^s + n\rho^w c_p^w \quad (6)$$

$$\chi_{eff} = (1-n)\lambda^s + n\lambda^w \quad (7)$$

where c_p^s and c_p^w are specific heats of solids and water respectively and λ^s and λ^w are thermal conductivity of solids and water respectively.

3. Boundary Value Problem

In present study, numerical simulation of one dimensional non-isothermal fully saturated consolidation problem is investigated, previously has been solved by Aboustit et al. [8] and Lewis and Schrefler [16] using finite element method.

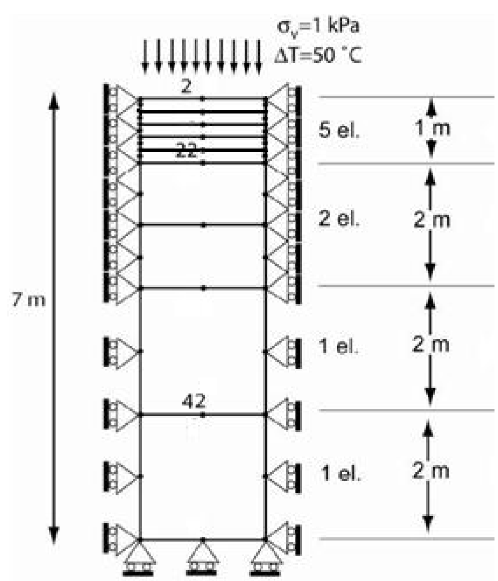


Fig. 1 Discretization and boundary conditions for non-isothermal consolidation problem [10]

A clay column of 7 m high and 2 m wide is subjected to an external compressive surface traction load of 1.0 kPa besides a thermal jump of 50°C above the initial ambient temperature. The soil is initially water saturated. Upper surface is considered drainage free but the lateral and bottom ones are completely insulated against drainage. Horizontal displacements are constrained along the vertical boundaries and vertical displacements are constrained at the bottom surface. The column is discretized using nine eight-node isoparametric elements as shown in Fig. 1.

All 48 nodes of elements are consequently numbered in each row from the left to the right. For example, nodes 7, 27 and 37 are the middle down nodes of the 1st, 5th and 7th elements on the figure. Furthermore, 2 × 2 Gauss integration points are used and plane strain condition is assumed. Material parameters used in numerical computation are listed in Table (1).

Table 1 Properties used in numerical simulation

Thermal conductivity of solids	Thermal conductivity of water	Specific heat of solids	Specific heat of water	Absolute viscosity	Thermal expansion coefficient of solids	Thermal expansion coefficient of water	Intrinsic permeability	porosity
$\lambda^s\left(Wm^{-1}{}^\circ C^{-1}\right)$	$\lambda^w\left(Wm^{-1}{}^\circ C^{-1}\right)$	$C_p^s\left(Jkg^{-1}{}^\circ C^{-1}\right)$	$C_p^w\left(Jkg^{-1}{}^\circ C^{-1}\right)$	$\mu^w\left(Nm^{-2}s\right)$	$\beta^s\left({}^\circ C^{-1}\right)$	$\beta^w\left({}^\circ C^{-1}\right)$	$K\left(m^2\right)$	n
0.20	0.57	732	4186	0.001	0.90E-6	0.35E-3	0.46E-16	0.2

4. Numerical Analysis

Numerical analysis performed in two different coupled froms, hydro-mechanical (HM) and thermo-hydro-mechanical (THM) using finite element program PISA [18, 19]. Comparison of these two forms shows the importance of considering thermal component of behavior and its effects on results of coupled analysis. Also in each form, two different kinds of analysis performed using linear elastic and elastoplastic constitutive models with two different preconsolidation pressures of 0.3 and 0.4 kPa. The results are determined considering the variations of temperature, pore pressure and vertical displacement with time. Results of analysis are compared with numerical solution of the same problem presented by Lewis and Schrefler [16] for verification which indicates the importance of applied constitutive model and its effects on results of analysis.

4.1. Hydro-mechanical (HM) analysis

At first, the problem has been simulated considering the loading in a coupled hydro-mechanical (HM) form using both linear elastic and elastoplastic constitutive models. Modified Cam clay model is used as the elastoplastic constitutive model and its parameters are shown in Table (2).

Table 2 Cam clay model and elastic parameters used in analyses

Initial void ratio	Slope of normal compression line	Slope of unloading/reloading line	Slope of critical state line	Preconsolidation pressure	Poisson ratio	Elastic modulus
e_0	λ	κ	M	$p_c(kPa)$	ν	$E(kPa)$
0.25	0.14	0.05	1.05	0.3 & 0.4	0.4	6000

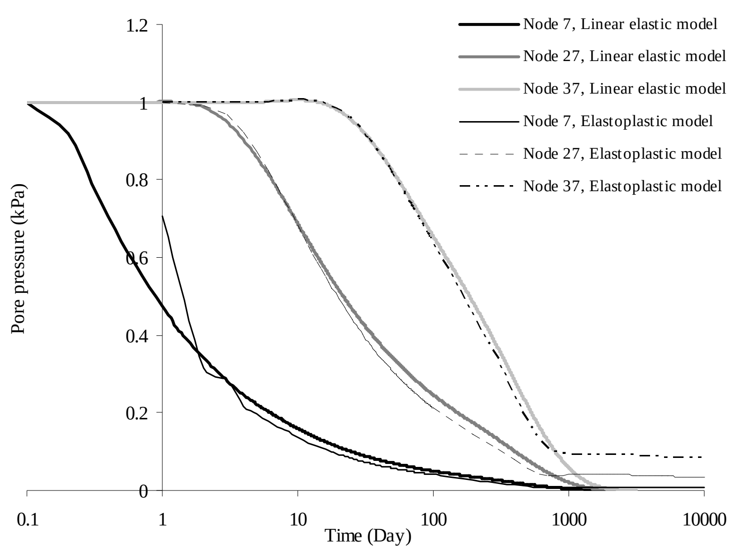


Fig. 2 Pore pressure variation versus time at three different nodes in hydro-mechanical analysis

Figure 2 depicts the history of pore pressure in three different nodes of soil column using linear elastic and elastoplastic constitutive models with preconsolidation pressure of 0.3 kPa. As the Figure shows, pore pressure increases to its ultimate value immediately after application of loading and dissipates gradually afterward as expected during mechanical consolidation. Also it dissipates more rapidly in nodes closer to the application point of mechanical loading. Application of both constitutive models in analysis yields nearly similar results.

Variations of vertical displacements with time are indicated in Fig. 3. According to the figure, vertical displacement of elastic analysis enhances by increasing time and remains constant at its maximum value. For the elastoplastic analysis with preconsolidation pressure of 0.3 kPa, the difference between elastic and elastoplastic analyses increases in longer times and vertical displacement does not reach to a constant ultimate value, however, increasing the preconsolidation pressure to 0.4 kPa, it reaches to a constant value in longer times similar to elastic analysis. This implies that preconsolidation pressure of 0.3 kPa is not an appropriate value and 0.4 kPa is the limit of elastic behavior in the considered problem. Finally, vertical displacement values are higher in nodes closer to the application point of loading.

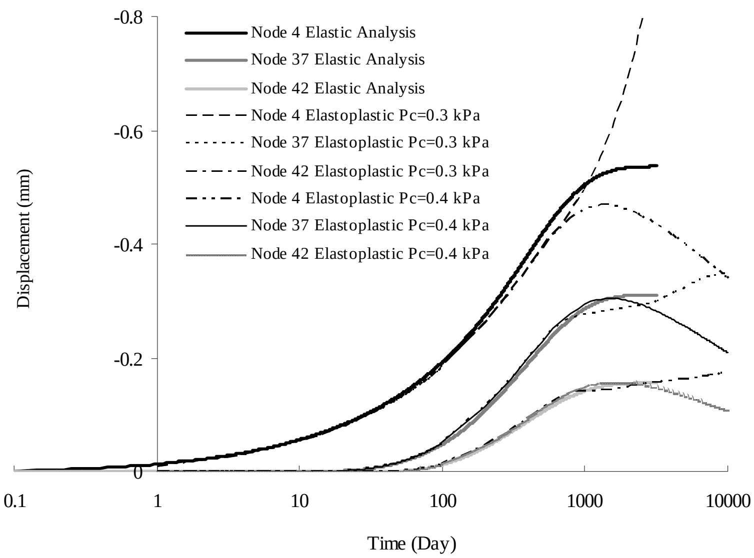


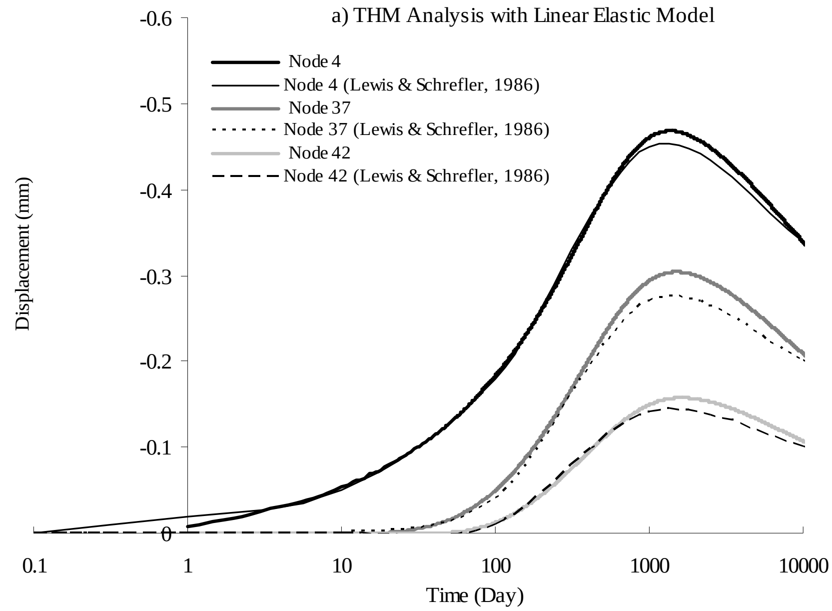
Fig. 3 Vertical displacement variation versus time at three different nodes in hydro-mechanical analysis

4.2. Thermo-hydro-mechanical (THM) analysis

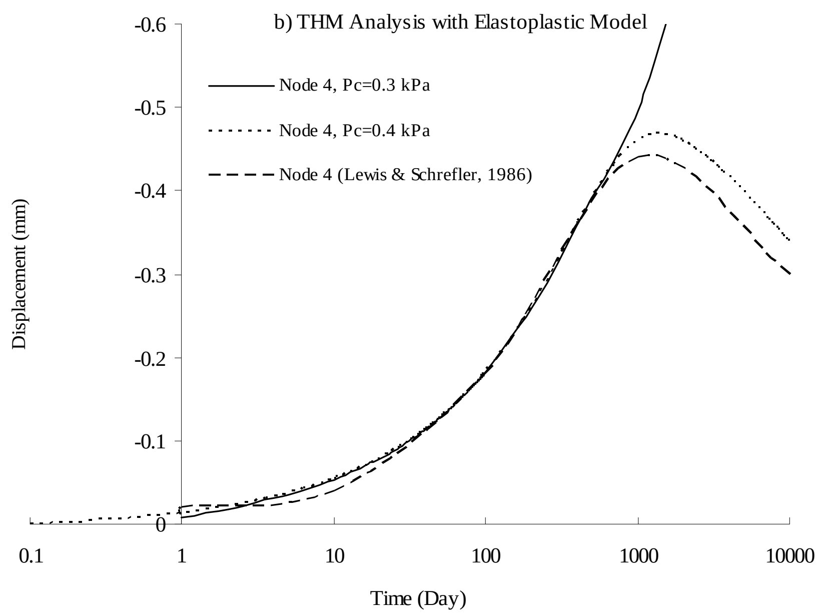
Coupled thermo-hydro-mechanical analysis of problem is also simulated to investigate the coupled phenomenon of thermal consolidation. Figures 4a and 5a depict the results for the history of vertical displacement and pore pressure using linear elastic model respectively. Also Figs. 4b and 5b depict the same results for the elastoplastic analysis. In each figure, results are compared and verified with numerical solution of Lewis and Schrefler [16]. They applied finite element method using isoparametric eight nodded elements. Material parameters are the same as present study as shown in Tables (1) and (2). Fairly good agreement can be observed with results of Lewis and Schrefler [16], however, it seems that results of vertical displacement are in better agreement with other numerical ones compared to the pore water pressure values. Maximum and minimum values are nearly identical and there is just a phase delay which can be attributed to the different time steps used in analysis.

During thermal consolidation, a tendency for expansion with temperature enhancement occurs which is usually balanced with void ratio reduction due to consolidation. As a result, an obvious peak point can be observed in [20] vertical displacement time history which is correctly simulated in Fig. 4 using both elastic and elastoplastic constitutive relations.

Figure 6 depicts the history of temperature changes for two different nodes according to thermo-hydro-mechanical analysis using elastic constitutive model. Results are also compared with analytical solution of Lewis and Schrefler [16] which indicates fairly good agreement. This implies that elastic constitutive relation can yield appropriate results for temperature changes during THM analysis.

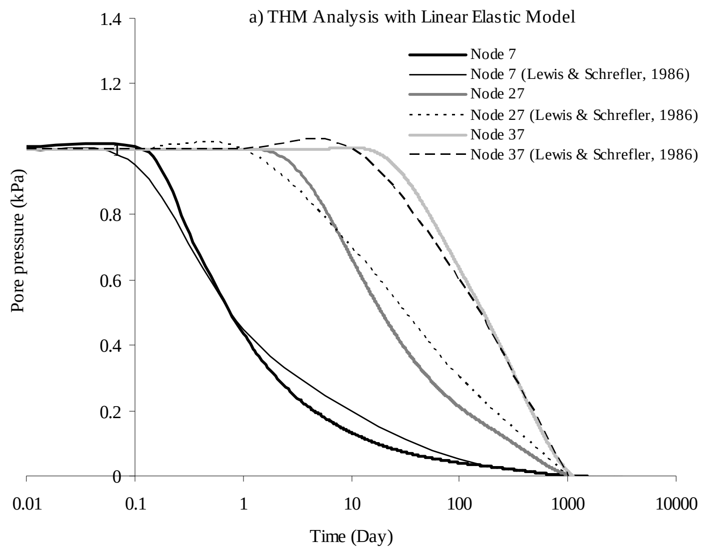


a) Linear elastic model

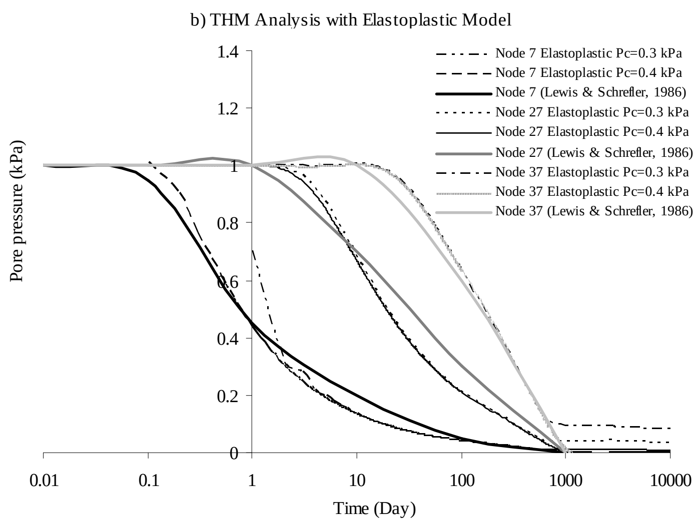


b) Elastoplastic model

Fig. 4 Comparison of vertical displacement values with analytical solution for THM analysis



a) Linear elastic model



b) Elastoplastic model

Fig. 5 Comparison of pore pressure values with analytical solution for THM analysis

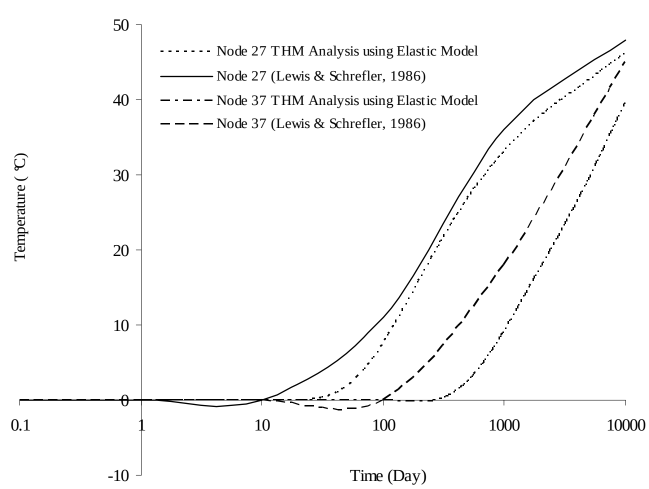


Fig. 6 Temperature variation versus time at nodes 27 and 37 during elastic THM analysis

4.3. Comparison of HM and THM analyses

Results of hydro-mechanical and thermo-hydro-mechanical analyses are compared in Figs. 7 and 8 using linear elastic constitutive model. As Fig. 7 indicates, trend of vertical displacements are the same in short times while the final values are different in longer times. In HM analysis, vertical displacement remains constant after reaching the maximum while in THM analysis it reduces after peak point to lower ultimate values. It can be related to the balance between thermal expansion of porous media and its subsequent compression due to consolidation. Figure 8 compares the trends of pore pressure variation for both HM and THM analyses. According to the figure, trend of pore pressure variation is nearly the same for both analyses. This implies negligible thermal effects on hydraulic behaviour of porous media.

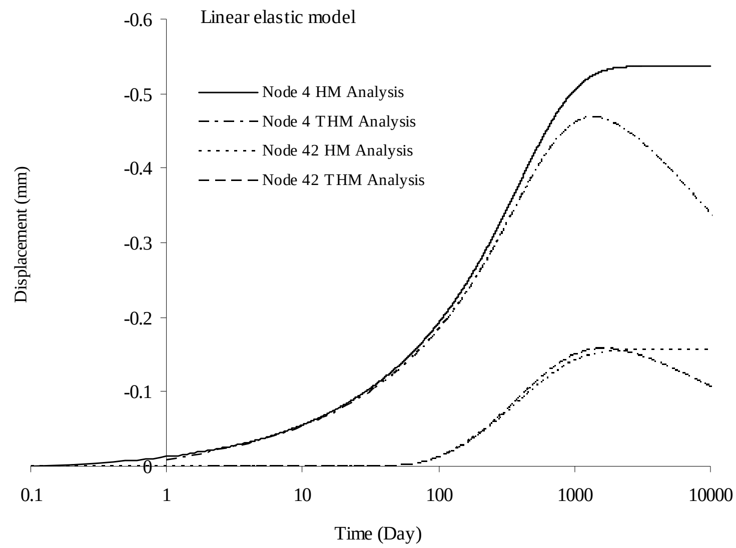


Fig. 7 Comparison of vertical displacement values obtained by HM and THM analyses

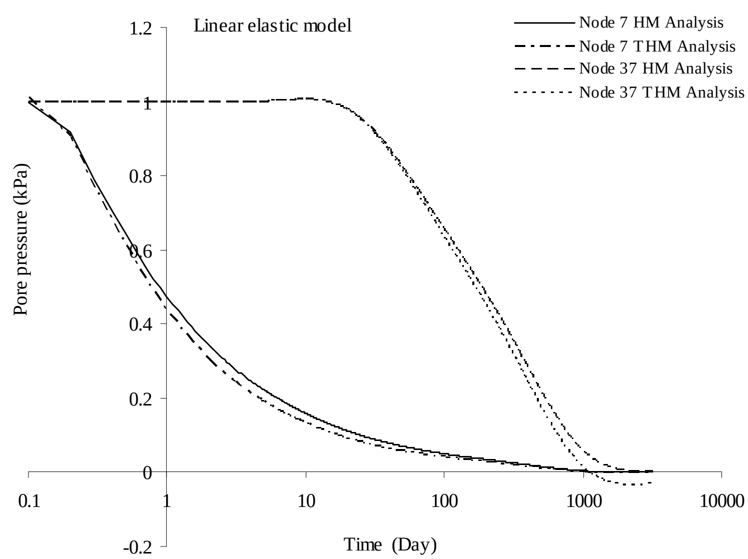


Fig. 8 Comparison of pore pressure values obtained by HM and THM analyses

5. Conclusion

In present paper, a numerical approach is used to investigate the effect of temperature on coupled thermal consolidation of water saturated porous media. To this end, elastic and elastoplastic constitutive models have been implemented in finite element code, PISA, for analysis of non-isothermal solid porous materials. Solution for one-dimensional consolidation of a clay column subjected to both vertical loading and uniform temperature is determined and pore water pressure dissipation besides vertical displacements subsequent to heat-pressure loadings are obtained.

Results show considerable difference in vertical displacement considering coupled THM analysis. Application of an elastoplastic constitutive model with correct preconsolidation pressure value affects the results and can be recommended instead of elastic one in THM analysis. Finally, it was determined that temperature effect on pore pressure variation is negligible during elastic analysis.

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