International Journal of Civil Engineering

Geotechnical Engineering

Evaluation of lateral spreading utilizing artificial neural network and genetic programming

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Received: 2012/12/26, Revised: 2013/08/22, Accepted: 2013/09/17

Abstract

Due to its critical impact and significant destructive nature during and after seismic events, soil liquefaction and liquefaction-induced lateral ground spreading have been increasingly important topics in the geotechnical earthquake engineering field during the past four decades. The aim of this research is to develop an empirical model for the assessment of liquefaction-induced lateral ground spreading. This study includes three main stages: compilation of liquefaction-induced lateral ground spreading data from available earthquake case histories (the total number of 525 data points), detecting importance level of seismological, topographical and geotechnical parameters for the resulted deformations, and proposing an empirical relation to predict horizontal ground displacement in both ground slope and free face conditions. The statistical parameters and parametric study presented for this model indicate the superiority of the current relation over the already introduced relations and its applicability for engineers.

Keywords: *Lateral spreading, Artificial neural network and genetic programming.*

1. Introduction

Once the liquefaction potential assessment suggests that the soil is likely to liquefy, it is critical to estimate the consequences that can be expected, in terms of ground movements or lateral displacements. During lateral spreading, the integral masses of surficial soil displace downslope or towards a free face along a shear zone that has formed within the liquefied soil layer. This displacement of the soil mass can vary from a few centimeters to several meters and cause considerable damage to engineered structures and lifelines. The amount of displacement due to lateral spreading depends on physical and mechanical characteristics of the soil layers at the site, water table depth, magnitude of earthquake, distance from the site to the energy source, ground slope conditions, thickness of the critical layer, and attenuation properties of the in situ soil. Therefore an engineering method is needed to estimate the aftermath ground movements.

In the recent few decades, a series of techniques, ranging from simple empirical relations to complicated numerical methods, have been proposed. This paper reviews the different assessment methods of ground movements due to lateral spreads, and then proposes an engineering relation, using a credible data set of case histories, to predict the value of lateral spreading.

2. Methods of analysis of lateral spread

The methods of analysis of ground displacements due to lateral spreading can be categorized in four groups: (i) simplified analytical methods (ii) numerical methods (iii) empirical relations based on case histories (iv) laboratory and centrifuge studies. These methods are summarized in Table 1 and discussed in the following.

2.1. Simplified analytical methods

2.1.1. Newmark's sliding block analysis

Newmark [26] introduced a method to analyze earthquake-induced sliding of slopes using sliding block model. This model estimates the displacement by double integration of acceleration when it exceeds the yield accelerations. Several relationships were proposed based on Newmark's technique [1 and 2], to estimate lateral spreading.

Scott et al. [3] back analyzed 39 well-documented

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Table 1 Summary of the methods of displacement assessment due to lateral spreading

Technique			Reference
Simplified Analytical method	Newark' sliding block		[1,2 and 3]
	Minimum potential energy model		[4 and 5]
Numerical method	Finite element method		[6, 7, and 8]
	Finite difference method		
Approaches based on case histories	Based on MLR method	Empirical method	[9, 10, 11, 12, 13 and 14]
		Semi-empirical method	[15, 16 and 17]
	Machine learning	Neural network	[18 and 19]
		Genetic programming	[20]
		Other methods	[21]
Laboratory and centrifuge studies	Laboratory		[22 and 23]
	Centrifuge		[24 and 25]

liquefaction-induced lateral spreads in terms of a mobilized strength ratio, su(mob)/σ′vo, using Newmark sliding block method. Based on the results of inverse analyses, they found that the back-calculated strength ratios mobilized during lateral spreads can be directly correlated to the normalized cone penetration test tip resistance and standard penetration test blow count.

In recent years, the sliding block model has been less accounted for evaluation of lateral spreads induced by liquefaction mainly because of the following difficulties:

• The task of determining the yield acceleration through finding the exact shear strength of a liquefiable soil is challenging in itself.

• Without an exact definition of slip surface, determination of yield acceleration would be complicated due to its changes during deformations following liquefaction.

• Occurrences of lateral spreading could continue after stopping the earthquake shaking which cannot be modeled by Newmark model.

Although there are some research studies indicating successful prediction of ground movement using Newmark model, this method has not been wildly used by engineers to predict ground movement caused by lateral spreading.

2.1.2. Minimum potential energy model

Towhata et al. [4] proposed an analytical model to estimate the displacements due to lateral spreads using minimum potential energy theory. In this model, the position of soil layers is determined based on minimum energy potential and Lagrangian equation of motion. They assumed a sinusoidal equation as a pattern for horizontal deformations of a vertical cross section in the liquefied deposit, while deforming volume of liquefied soil was assumed to be unchanged.

Tokida et al. [5] proposed simplified version of this method. In their model, the maximum of lateral displacement takes place at the center of the block which the value of displacement can be easily predicted. This method has also following major disadvantages:

• It is not possible to model the steep free faces, usually presented in the field.

• The model is only capable of predicting the maximum or ultimate displacements.

This model is also only capable of predicting the maximum or ultimate displacements and has been less implemented by engineers compared with Newmark sliding block model.

2.2. Numerical Method

In order to model all the details of liquefaction- induced ground movement, a proper numerical model should be able to simulate seismic excitation, softening of the soil due to increase in pore water pressure, rapid decrease in shear strength, continuation of displacement after dynamic loading, and reconsolidation caused by drainage of excess pore pressure. Hamada et al. [27] were among the first researchers to evaluate ground displacement induced by liquefaction, using finite element technique. They employed 2D finite element technique to model a lateral spread in Noshiro, Japan. However, their model was highly dependent on the assumption of elastic behavior for the material. Using numerical methods, other researchers have made different assumptions for modeling the lateral spreads. An advanced analysis was performed by Finn et al. [7 and 8], using TARA-3, and TARA-3FL code. Yasuda et al. [28] also introduced a very simple finite element technique. In summary, varieties of simplified finite element schemes have been used to model liquefaction and lateral spreading. Numerical studies have shown limited success when compared to observed field behavior. Numerical methods have not been popular for the estimation of deformation.

2.3. Empirical methods (Approaches based on case histories)

These methods have been used extensively by engineers because of simplicity. The engineers use these simple methods vastly. As shown in Table 1, two methods can be designated to estimate the lateral spreading using empirical relations:

2.3.1. MLR method (empirical and semi – empirical method)

There are empirical and semi-empirical methods available to estimate lateral displacement induced by liquefaction. The Multi Linear Regression (MLR) method, developed based on regression of collected data has been implemented by many researchers such as Bardet et al. [11, 12 and 13], Youd et al. [14], Shamoto et al. [15], Zhang et al. [16], Zhang and Zhao [17], Hamada et al. [27], Rauch and Martin [10] and Youd and Perkins [29].

Hamada et al. [27] proposeda simple empirical relationship for estimating lateral spreads, based on 60 data sets, most of them collected from the Noshiro earthquake.As can be seen in Table 2, this equation is very simple and easy to use. The deficiency for this approach is that it has been proposed based on a very limited number of data. The Hamada et al. [27] method suggests that only two parameters from the site geometry affects the value of lateral spreading, and other geotechnical and seismic parameters are not accounted. As a result, this equation is not general enough to be used for other sites.

Youd and Perkins [29] proposed a simple relationship based on the data collected from western United States and Alaska historical earthquake records. The value of displacement (LSI)

Table 2 Summary of the proposed equations to evaluate ground displacements

REFERENCE	EQUATION	REMARK
[27]	$D_h - 0.75\sqrt{H}\sqrt[3]{-}$	Dh is the predicted horizontal ground displacement (m), H is the thickness of liquefied zone (m), and θ is the larger slope of either ground surface or Liquefied zone lower boundary (%).
[29]	$logLSI - -3.49 - 1.86logR - 0.98M_W$	LSI is the general maximum amplitude of lateral ground spread (inch), R is the horizontal distance to the seismic energy source (km), and Mw is the earthquake moment magnitude
[1]	$D - N_{eq} - T^2 \; a_P - f \; -a_y / a_P -$	D is the lateral ground deformation, $N_{eq}$is the number of cycles equivalent to uniform base motion, T is the period (s), a_y is the yield acceleration (g), a_p is the peak acceleration (g), and f is a dimensionless function depending on base motion
[2]	$D - N_{eq} - V_{max}{}^2 / a_P - f \; -a_y / a_P -$	V_{max} is the peak velocity, recommended N_{eq}=2
[5]	$For 10m - L - 100$ $D - 1.73-10^{-5} L^{1.94} H^{0.298} T^{-0.275} \_^{0.963}$ $\text{For} 100m - L - 1000$ $D - 1.29-10^{-5} L^{1.99} H^{0.28} T^{-0.243} \_^{0.995}$	D is the horizontal displacement (m), L is the length of the slide (m), H is the average thickness of the liquefied layer (m), T is the average thickness of the Liquefied surface layer (m), and θ is the slope of ground surface (%).
[11, 12]	**For combined free-face and ground slope model:(for 6 parameters)** $\log - D_H - 0.01 - - -15.034 - 1.096M - 0.873\log R - 0.014R - 0.634\log W - 0.275\log S - 0.494\log T_{15}$ $-4.053\log - 100 - F_{15} - -0.814D_{5015}$ **For free-face model:(for 6 parameters)** $\log - D_H - 0.01 - - -17.372 - 1.248M - 0.923\log R - 0.014R - 0.685\log W - 0.3\log T_{15} - 4.826\log - 100 - F_{15} -$ $-1.091D_{5015}$ **For ground slope model:(for 6 parameters)** $\log - D_H - 0.01 - - -14.152 - 0.988M - 1.049\log R - 0.011R - 0.318\log S - 0.619\log T_{15} - 4.287\log - 100 - F_{15} -$ $-0.705D_{5015}$	
[14]	**For free-face model** $\log - D_H - 0.01 - - -17.372 - 1.248M - 0.923\log R^* - 0.014R - 0.685\log W - 0.3\log T_{15} - 4.826\log - 100 - F_{15} -$ $-1.091D_{5015}$ **For ground slope model:** $\log - D_H - 0.01 - - -14.152 - 0.988M - 1.049\log R^* - 0.011R - 0.318\log S - 0.619\log T_{15} - 4.287\log - 100 - F_{15} -$ $-0.705D_{5015}$ $(R^* - R - 10^{-0.89M - 5.64 -})$	
[20]	**For free-face model** $Dh - -234.1\frac{1}{M^2 - R \; W} - 156\frac{1}{M^2} - 0.008\frac{F_{15}}{R^2 - T_{15}} - 0.01\frac{W - T_{15}}{R} - 2.9\frac{1}{F_{15}} - 0.036\frac{M - T_{15}{}^2 D50_{15}{}^2}{R^2 - W} - 9.4\frac{M}{R - F_{15}} - 4-10^{-6}\frac{M - R^2}{D50_{15}} - 3.84$ **For ground slope model:** $D_h - -0.027\frac{T_{15}{}^2 - F_{15}}{M^2} - 0.05\frac{R - T_{15}}{M^2 - D50_{15}} - 0.44\frac{1}{M - R^2 \; S \; T_{15}} - 0.03R - 0.02\frac{M}{S - T_{15}} - 5-10^{-5}\frac{M - R^2}{D50_{15}{}^2} - 0.075M^2 - 2.4$	

is estimated based on the distance to energy source of the earthquake and Moment Magnitude (M) with the upper limit of 100 inches. This model assumes that if the liquefaction occurs and it causes lateral spreading, the amount of ground displacement depends on only seismic parameters (R, M). This equation had attracted the attention of engineers at its time. While this equation might have been suitable for estimating the lateral spreads in the western part of USA but it also lacked enough generality and so it did not become popular worldwide.

Rauch and Martin [10] considered liquefaction-induced lateral spreading as slides of finite area instead of individual displacement vectors. Using multiple linear regression methods, Rauch proposed three different equations for estimating the average lateral deformations, which were referred to as regional, site, and geotechnical equations. This model was based on the MLR analysis of a total of 78 data points from 16 different earthquakes. The methodology of this model was to subdivide the liquefied area into separate slide zones, to define the length and area for each slide, and then to consider the average liquefaction-induced displacement vectors and the average borehole soil properties within these slides. The quality of the fitted results for the three equations of this MLR model was reported as R^2=50.9% based on 71 data points for the regional model, R^2=67.0% based on 58 data points for the site model, and R^2=68.8% based on 45 data points for the geotechnical model.

Shamoto et al. [15] employed laboratory-based estimates of liquefaction-induced limiting shear strains coupled with an empirical adjustment factor to relate these laboratory values to the observed field behavior. The predicted lateral displacements, based on laboratory limiting shear strains, are multiplied by a factor of 0.16 to predict the lateral displacements of non-sloping ground.

Bardet et al. [11, 12 and 13] proposed six relationships with four and six parameters using the data reported by Bartlett and Youd [33 and 34]. The equation with four parameters is suitable for conditions with limited borehole data, while the equation with six parameters is more accurate than that with four parameters.

The model of Youd et al. [14], originally developed in 1992, was derived using MLR method for the data collected from earthquakes in Japan and USA. The model has two different equations for free face and ground slope (Table 2).

Because of using wide range of data from different earthquakes and also employing three parameters of; geometry of the site; geotechnical data; and the characteristics of earthquake, this model could find more popularity among the geotechnical engineers. However, there are still some limitations in its application. For instance, the free face equation is used when $5 \leq W \leq 20\%$, while the ground slope equation is valid when $W \leq 1\%$. The discontinuous bordering system for the values of W provides no explanation for the cases with $1<W<5\%$.

It should be also noticed that, the estimations made by this model for Chi Chi earthquakes in Taiwan (Chu et al. [30]) and Kocaeli (Cetin et al. [31] and Youd et al. [32]) earthquake in Turkey are not practically applicable for the observed lateral spreads in the sites.

Bardet et al. [11 and 12] proposed six relationships with four and six parameters, using the data reported by Bartlett and Youd [33 and 34]. The equation with four parameters is suitable for conditions with limited borehole data, while the equation with six parameters is more accurate than the one with four parameters. The models proposed by Youd et al. [14] and Bardet et al. [11 and 12] models are the most popular methods used by engineers worldwide.

Zhang et al. [16] predicted a semi empirical approach to estimate liquefaction-induced lateral displacements using standard penetration test (SPT) or cone penetration test (CPT) data. Their approach combines the available SPT- and CPT-based methods to evaluate the liquefaction potential with laboratory test results for clean sands to estimate the potential maximum cyclic shear strains for saturated sandy soils under seismic loading. A lateral displacement index (LDI) is then introduced, which is obtained by integrating the maximum cyclic shear strains with depth.

2.3.2. Machine learning technique

For complex problems where the relationship between the variables is unknown, the machine learning technique (for example artificial neural network (ANN) or Genetic Programming (GP), etc) is a powerful predictive tool, as long as it resembles the nature of the situation.

Other researchers have previously shown that the complex phenomena such as liquefaction have been predicted more accurately by ANN than by the conventional methods (Goh [35], Baziar and Nilipour [36]).

While employing Machine learning approach, the following points should be taken into account:

1. The approach should be generalized such that new available data, not included in the model, may not contradict the model.
2. It should be noticed that not any derived equations, even with a good degree of accuracy, is acceptable. The resulted equation must be physically compatible with the phenomenon. The parametric study of the model can help to find out the compatibility between the proposed model and the real phenomenon.
3. The relation should be based on a comprehensive statistical analysis and it is not reasonable to judge its accuracy through R^2 or RSME only. In other words, different statistical parameters should be considered in order to judge the accuracy of the proposed relation.
4. For machine learning technique, the available data is usually divided into two parts of training and testing sets. Firstly, there should be logical reasoning for division of the data into training and testing sets so that each set represents the whole range of data. Secondly, the final relation should be accepted if there is not much difference between testing and training statistical parameters.
5. The number and the range of data should be such that the proposed equation is credibility valid and general enough. It means that the number of data provides an influential effect on the machine learning analysis and the resulted model.

Wang and Rahman [18] developed a model to predict the horizontal ground displacements using aback-propagation neural network analysis. A database containing the case histories of lateral spreads observed in eight major earthquakes (367 cases) was used. The results of their study indicated that theneural network model served as a reliable and simple predictive tool for the amount of horizontal ground displacement. As additional data become available, the model itself can be improved to makemore accurate displacement predictions for a wider range of earthquake and site conditions.

Baziar and Ghorbani [19] developed a model to predict the horizontal ground displacement inboth ground slope and free face conditions due to liquefaction-induced lateral spreading using aneural network analysis. They investigated the influence of the seismological, topographical andgeotechnical parameters on the resulting deformations and their degrees of importance based on 484cases from 11 sites with the highest degree of accuracy (R^2=92%). Their sensitivity analysis indicatedthat the two factors of source distance (R) and mean grain size ($D50_{15}$) have the most significant effect on the predicted displacements.

Javadi et al. [20] presented a model based on GP for determination of liquefaction-induced lateral spreading. The GP models were trained and validated using a database of SPT-based case histories.

Separate models were presented to estimate the lateral displacements for free face and for gently sloping ground conditions. They compared their GP models with the multi linear regression (MLR)model and highlighted the advantages of the proposed GP model over the conventional methods.

Oommen and Baise [21] presented a machine learning technique known as support vector regression (SVR) using free face lateral displacement data. They claimed that SVR has relatively better predictive capability than the commonly used empirical relationship using multi linear regression.

2.4. Centrifuge study

Sharp et al. [24] and Kutter et al. [25] numerically analyzed the centrifuge model experiments data to investigate the average shear and volumetric strains for the seismically liquefied soils. The related major effective engineering factors were thickness of liquefied soil sub-layer and soil relative density.

Physical model testing such as centrifuge and shaking table test have shed light on the understanding of complex problems such as lateral spreading induced by liquefaction. A summary of the different equations presented by researchers to estimate lateral spreading is given in Table 2.

3. Development of a new model

According to the points above and disadvantage of methods, a new empirical model is needed to estimate lateral spreads with the following characteristics:

- Compatible with the concept of phenomenon.
- Able to cover the possible range of ground geometry without any gap.
- Better accuracy

3.1. Data set and statistical parameters

Many researchers have widely accepted that site topography, soil characteristics and earthquake characteristics are the required parameters for any lateral spreading analysis. However, each researcher may use different parameters as representative of each category of parameter. Youd et al. [14] have compiled the most complete data bases (484 data) up to 2002. In current study, new 41 data (Table 3) from Chi-chi earthquake, Taiwan (1999) and Kocaeli earthquake, Turkey (1999) are added to the above data set. Therefore a total of 525 data were used in current study. Summary of the new 41 earthquake events used in current study is presented in Table 4.

It has been widely accepted, among the researchers, that the input parameters selected by Youd et al. [14] is a complete and suitable set of input parameters to control the lateral spreading. For this reason, the same parameters have been selected as the effective parameters, by many other researchers as well as in the current study. These parameters are: the moment magnitude of the earthquake (M), thenearest distance to the seismic energy source (R), thecumulative thickness of saturated granular layers withcorrected blow counts of SPT less than 15 (T_{15}), theaverage fines content for granular materials includedwithin T_{15} (F_{15}), the average mean size for granularmaterials within T_{15} ($D50_{15}$), the ground slope (S) and the free-face ratio (W). The range of above parameters, for the total of 525 data, is presented in Table 5.

Despite of broad range of the historical data of lateral spreads collected and employed for this study, it should be noted that these data do not cover all the possible situations; thus, special attention should be paid to the values of input parameters, while using the proposed equation.

Adding of 41 new data to the Youd's data, resulted in the data included from more countries, made the range of topography (W, S) parameters broader for which the maximum amount of slope ground increased from 11 to 17 %.

The following statistical parameters were used in order to assess the quality and accuracy of the proposed equation compared with previous relations:

The coefficient ofdetermination (R^2), while different equations are available to obtain (R^2), the following equation, because of its broad popularity, was implemented in this study.
root means squared error (RMSE),

Coefficient of correlation (R) and

Mean absolute error (MAE). These parameters are defined as:

$$R^2 = \left(1 - \frac{\sum_{i=1}^{n}(M_i - P_i)^2}{\sum_{i=1}^{n}(P_i)^2}\right) \times 100\%;$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(\mathbf{M_i} - \mathbf{P_i})^2}{\mathbf{n}}};$$

$$MAE = \left(\frac{1}{n}\sum_{i=1}^{n}\left|\frac{M_i - P_i}{M_i}\right|\right) \times 100\%;$$

$$R = \frac{n\sum_{i=1}^{n}(M_iP_i) - \sum_{i=1}^{n}(M_i)\ \sum_{i=1}^{n}(P_i)}{\sqrt{n\sum_{i=1}^{n}(P_i^2) - (\sum_{i=1}^{n}(P_i))^2}\sqrt{n\sum_{i=1}^{n}(M_i^2) - (\sum_{i=1}^{n}(M_i))^2}} \times 100\%$$

Where; M_i, P_i and n are measured, predicted and number of cases, respectively.

Table 3 The 41 new data used in the current study

EARTHQUAKE	Mw	R	W	S	T_{15}	F_{15}	$D50_{15}$	Observation	References
KOCAELI- TURKEY 1999		35	20	0	4.2	19	0.23	0.2	
KOCAELI- TURKEY 1999		35	13	0	3.6	20	0.21	0.15	
KOCAELI- TURKEY 1999		35	8	0	5.7	18	0.2	0.05	
KOCAELI- TURKEY 1999		0.5	5	17	0.00	N.A[a]	N.A	0.00	
KOCAELI- TURKEY 1999		0.5	20	17	0.8	20	2.9	0.9	
KOCAELI- TURKEY 1999	7.5	0.5	15	0	1.4	52	0.074	1.2	[31 and 32]
KOCAELI- TURKEY 1999		0.5	7	0	2.2	41	1.3	0.3	
KOCAELI- TURKEY 1999		0.5	8.00	1	1.20	11	7.7	0.9	
KOCAELI- TURKEY 1999		0.5	6.00	1	1.70	31	0.55	0.1	
KOCAELI- TURKEY 1999		0.5	8.00	10	2.70	12	1.6	2.4	
CHI-CHI, TAIWAN1999		13	16.20	3.8	1.70	22.3	0.12	0.25	
CHI-CHI, TAIWAN1999		13	5.90	3.8	1.70	22.3	0.12	0.05	
CHI-CHI, TAIWAN1999		5	23.00	0.2	1.80	48.5	0.1	1.62	
CHI-CHI, TAIWAN1999		5	13.30	0.2	1.80	48.5	0.1	0.62	
CHI-CHI, TAIWAN1999		5	11.70	0.2	1.80	48.5	0.1	0.52	
CHI-CHI, TAIWAN1999		5	9.60	0.2	1.80	48.5	0.1	0.35	
CHI-CHI, TAIWAN1999		5	31.30	0.2	1.00	31.4	0.1	2.96	
CHI-CHI, TAIWAN1999		5	19.00	0.2	1.00	31.4	0.1	0.96	
CHI-CHI, TAIWAN1999		5	10.50	0.2	1.00	31.4	0.1	0.61	
CHI-CHI, TAIWAN1999		5	8.00	0.2	1.00	31.4	0.1	0.35	
CHI-CHI, TAIWAN1999		5	57.70	0.2	0.45	30	0.13	1.240	
CHI-CHI, TAIWAN1999		5	24.60	0.2	0.45	30	0.13	1.0	
CHI-CHI, TAIWAN1999		5	14.30	0.2	0.45	30	0.13	0.65	
CHI-CHI, TAIWAN1999		5	12.20	0.2	0.45	30	0.13	0.400	
CHI-CHI, TAIWAN1999		5	26.30	0.125	1.10	20.8	0.11	0.01	
CHI-CHI, TAIWAN1999	7.6	5	11.90	0.125	1.10	20.8	0.11	0.010	[30]
CHI-CHI, TAIWAN1999		5	4.80	0.125	0.00	N.A	N.A	0.0	
CHI-CHI, TAIWAN1999		5	21.20	0.3	0.75	13	0.18	0.49	
CHI-CHI, TAIWAN1999		5	15.00	0.3	0.75	13	0.18	0.29	
CHI-CHI, TAIWAN1999		5	9.00	0.3	0.75	13	0.18	0.23	
CHI-CHI, TAIWAN1999		5	7.90	0.3	0.75	13	0.18	0.17	
CHI-CHI, TAIWAN1999		5	6.60	0.3	0.75	13	0.18	0.1	
CHI-CHI, TAIWAN1999		5	5.70	0.3	0.50	13	0.18	0.01	
CHI-CHI, TAIWAN1999		5	3.50	0.3	0.00	N.A	N.A	0.0	
CHI-CHI, TAIWAN1999		5	49.90	0.125	0.80	20.8	0.11	2.05	
CHI-CHI, TAIWAN1999		5	37.30	0.125	0.80	20.8	0.11	1.05	
CHI-CHI, TAIWAN1999		5	25.20	0.125	0.80	20.8	0.11	0.8	
CHI-CHI, TAIWAN1999		5	18.40	0.125	0.80	20.8	0.11	0.55	
CHI-CHI, TAIWAN1999		5	13.70	0.125	0.80	20.8	0.11	0.45	
CHI-CHI, TAIWAN1999		5	7.40	0.125	0.50	20.8	0.11	0.01	
CHI-CHI, TAIWAN1999		5	5.30	0.125	0.00	N.A	N.A	0.0	

3.2. Machine learning technique for estimation of lateral spreading

3.2.1. Artificial neural network models

A neural network is a massively parallel distributed processor that has a natural propensity for storing experiential knowledge and making it available for use. It resembles the brain in two respects:

• Knowledge is acquired by the network through a learning process.

• Interneuron connection strengths known as synaptic weights are used to store the knowledge.

The procedure used to perform the learning process is called learning algorithm, a function to modify the synaptic weights of the network in an orderly fashion so as to attain a desired design objective.

In this study, the artificial neural network is employed to check the importance of each input parameter.

In this research, 20 percent of the data were randomly selected for testing and the remaining 80 percent were selected for training.

Table 4 The earthquake data used in the proposed model

No. of Site	Country	Year	Name of Earthquake	M	No. of Data	%
1	United States	1906	San Francisco	7.9	2	0.4
2	Alaska	1964	Prince William Sound	9.2	7	1.3
3	Japan	1964	Niigata	7.5	299	57.0
4	United States	1971	San Fernando	6.4	23	4.4
5	United States	1979	Imperial valley	6.6	31	5.9
6	Japan	1983	Nihonkai-Chubu	7.7	72	13.7
7	United States	1983	Borah Peak-Idaho	6.9	4	0.8
8	United States	1987	Superstition Hills	6.6	6	1.1
9	United States	1989	Loma prieta	7	2	0.4
10	Japan	1995	Kobe	6.9	19	3.6
11			Ambrasys's data	6.4-7.8	19	3.6
12	TURKEY	1999	Kocaeli	7.5	10	5.9
13	TAIWAN	1999	Chi-Chi	7.6	31	1.9

Table 5 Ranges of input and output parameters

input and output variables	Mean	Range
M=Earthquake magnitude	7.4	6.4-9.2
R= Distance from site to energy source, km	20.1	0.2-100
W=Free face ratio (%)	6.3	0-57.7
S=Ground slope (%)	0.99	0-17
T_{15}=Cumulative thickness of saturated granular layers(m)	7	0-19.7
$D50_{15}$=the average mean grain size(mm)	0.4	0.036-12
F_{15}=the average fines content (%)	13.8	0-70
D_h=measured lateral displacement(m)	2	0-10.2

After data division, it is important to pre-process the data into a suitable form before they are applied to the ANN. Pre-processing the data, such as scaling, is important to ensure that all variables receive equal attention during training. The output variables have to be scaled to be commensurate with the limits of the transfer functions used in the output layer. Scaling the input variables is not necessary but is always recommended. In this study, the input and output variables are scaled between 0.0 and 1.0 because a sigmoidal transfer function is used in the output layer. At the beginning of the training process, random values were used as weights. Because the initial values play an important role in this procedure, the training process has been conducted several times, each time with different random synapses values to reach results with the least errors. The Root Mean Square Error (RMSE) is the criterion to determine the errorof the resulting function.

Table 6 shows the weights of the input-hidden layer connections for the combination of free face and gentle slope model. Taking into account the connection weights, by partitioning the hidden-output connection weights into components connected with each input variable, one can assess the relative importance of various input factors and also regenerate the ANN model.

The relative importance of the each parameter obtained for the optimal ANN model (with one hidden layer and 12

Table 6 Connection weights of the ANN model

Hidden neurons	Connection weights						
	M	R	W	S	T_{15}	F_{15}%	$D50_{15}$
1	-0.0364	0.1819	0.0034	0.0416	0.0567	0	0.0093
2	-0.2102	0.5046	0.0668	0.0244	0.0108	0.0043	0.0892
3	-0.0074	-0.0373	0.1205	0.1103	-0.2494	-0.2932	0.1591
4	0.0217	-0.0284	0.0259	0.1454	-0.1163	-0.2516	0.2126
5	-0.1801	0.4327	-0.012	0.0711	-0.0091	-0.0307	0.1068
6	0.3164	0.5524	-0.3402	0.9905	0.7138	0.1572	0.599
7	-0.8349	1.4822	0.4356	-0.2432	-0.427	0.1752	-0.4827
8	-0.0608	0.1425	-0.0506	-0.9162	-0.2054	-0.1946	0.0492
9	-0.0767	0.3568	0.112	-0.1911	0.1944	0.1094	-0.0115
10	0.0731	0.0549	0.0834	-1.4277	-0.0965	-0.1455	0.1832
11	-0.0884	0.2365	0.0537	-0.1638	0.0851	0.0453	0.028
12	0.0352	0.4414	-0.2632	0.3346	0.0933	-0.1256	0.0855

neurons) is summarized in Table 7. The $D50_{15}$ and R parameters are more important than the other parameters. However, when the input parameters are categorized as earthquake, topography, and soil parameters, all of the studied parameters have noticeable importance, indicating that all of the parameters have effects on the lateral spreading. The obtained relative importance in this study is compatible with the same results, reported by Baziar and Ghorbani[19].

The performance of the neural network obtained in this study is compared with the other methods in Table 8. The results indicate that the ANN model presented in this study performed very well, with R^2 and RMSE of 0.92 and 0.67 m for training and 84.3 and 0.85 m for testing, respectively. Table 8 also shows that the results of testing are generally consistent with those obtained during training, indicating that the model is able to generalize within the range of data used for training.

3.2.2. Genetic algorithm and genetic programming

As an optimization technique, genetic algorithm (GA), which was evolved from the principles of genetics and natural selection, tries to search the minima of a given function using a trial process. Genetic algorithm optimizes an array of input variables or chromosome sin different types such as binary strings (0, 1), real strings (0, 1... 9), and representation of tree (computer programs). Koza developed a special genetic algorithm known as “genetic programming (GP)”in which each chromosome in the population is a program comprised of random mathematical functions and terminals. A function set could contain functions such as basic mathematical operators (+, -, n, /, etc.), Boolean logic functions (AND, OR, NOT, etc.), or any other user defined function (Cabalar and Cevik [37],Jafarian et al. [38], Askari et al. [39] and Khan [40]).

Based on the result of ANN, presented in this study, it is well understood that the lateral spreading is affected by all input parameters (M, R, T_{15}, F_{15}, $D50_{15}$, S and W)chosen originally by Bartlett and Youd [34].Therefore, the same seven input parameters were employed in the current study.

In order to develop the GP model, 20 percent of cases (105 cases), included the new 41 cases, were selected for testing and the remaining 80 percent (420 cases) were selected for training. The data division process was performed so that the main statistical parameters of the training and testing subsets became close to each other. To reach this goal, a trial selection procedure was carried out and most possible consistent division was determined. It is essential that the data used for training and testing represent the same population. In order to achieve such desired population in the present study, several random combinations of the training and testing sets were tried until two statistically consistent data sets were obtained. The mean, standard deviation and range of each input parameters used in the current study are presented in Table 9.

3.3. Result and discussion

Many data sets were executed with various initial setting, and the performances of the obtained equations were benchmarked. Selection of the best model was based on statistical criteria including: R^2, Root Mean Square Error (RMSE) and Mean Absolute Error (MAE). Moreover, a comprehensive parametric study was performed to monitor the behavior of model versus variations of input variables and also its compatibility with the physical concept of the phenomenon. The following relationship was finally selected as the best model for prediction of lateral spreading:

$$D_h = \frac{54.36\,T_{15}}{D_{50}+0.6532} - \frac{55.34\,T_{15}}{D_{50}+0.6689} + \frac{196.9\,T_{15}}{W+0.9212} - \frac{199.8\,T_{15}}{W+0.9434} + \frac{0.0446(W-S)}{R} - \frac{1.718}{S+0.8956}$$

$$- 0.02452T_{15}F_{15} - 0.00625F_{15}S + 0.001474R(W - T_{15}) - 0.06875T_{15}(W - S) + M(0.1058T_{15} + 0.009652T_{15}W - 0.1225) + 0.00024T_{15}F_{15}^2 - 0.00255RWS + 2.6 \quad \text{(Eq.1)}$$

Table 7 Relative importance of ANN input variables

Categories	Earthquake		Topography		Soil		
Factors	**M**	**R**	**S**	**W**	**T_{15}**	**$D50_{15}$**	**F_{15}**
Relative importance (%)	14.1	24	8.2	10.8	10.3	23.6	9
Total relative Importance (%)	38.1		19		42.9		

Table 8 The statistical parameters for various models

Model	number of data	Correlations coefficient (R^2)%			Root-Mean-Square Error RMSE,(m)		
		Total	**Testing**	**Training**	**Total**	**Testing**	**Training**
ANN 1999 (Wang and Rahman)	484	81.6	65	73.3	NR	NR	NR
ANN 2005 (Baziar and Ghorbani)	484	95	85	92	0.7	1.1	0.52
MLR-2002 (Youd et al.)	484	NA	NA	83.6	0.97	NA	NA
MLR-2002 (Youd et al.)	525	NA	NA	71.2	1.4	NA	NA
MLR-1999 (Bardet et al. 6 parameters)	525	NA	NA	69	1.4	NA	NA
ANN 2013 (Current study)	525	92.7	84.3	91.02	0.7	0.85	0.67

Table 9 Statistical properties of training and testing data sets used in current study to develop the GP-based model

Input and output variables	Subset	statistical parameters		Range
		Mean	Stddev	
M	Training Set	7.4	0.44	6.4-9.2
	Testing Set	7.4	0.41	6.4-9.2
R (km)	Training Set	20.6	11.85	0.2-100
	Testing Set	18.16	20.24	0.2-100
W (%)	Training Set	5.55	7.78	0-56.8
	Testing Set	10.88	9.41	1-57.5
S (%)	Training Set	0.94	1.04	0.05-11
	Testing Set	1.2	2.57	0-17
T_{15} (m)	Training Set	7.4	4.28	0.5-16.7
	Testing Set	5.43	5.09	0-19.7
F_{15} (%)	Training Set	12.5	11.89	0-66
	Testing Set	18.88	15.5	0-70
$D50_{15}$ (mm)	Training Set	0.38	0.66	0.06-12
	Testing Set	0.5	1.26	0.036-10
D_h (m)	Training Set	2.23	1.78	0.01-10.2
	Testing Set	1.23	1.34	0-7.13

Where, displacement value is assumed zero if $D_h<0$.

To perform a parametric study, other parameters were kept constant at their average values in the data set (reported in Table 5) while changing the parameter under consideration and evaluating the predicted displacement.

Fig. 1 (a-g) shows the predicted values of D_h as a function of each input parameter when the other parameters are set to their average. The results, in general, agree with the finding of previous researchers (Wang and Rahman [18], Youd et al. [14], Baziar and Ghorbani [19], etc.).

As it is shown in Fig. 1, increasing the values of M, T_{15}, F_{15}, $D50_{15}$, S and W parameters led to increase in the displacement values. The increasing F_{15} caused D_h to decrease, but the displacement value became zero for $F_{15}>23$. As indicated in Fig.1, no lateral spreading happens while other parameters are kept constant at their average values. In other words, the value of limiting $F_{15}=23$ would be changed with selecting different values for other parameters.

The predicted displacements for changes in $D50_{15}$, when other parameters are set to their average values, are plotted in Fig. 1-g. The authors believe that, to evaluate the effect of $D50_{15}$ on the displacement, the combination of three soil parameters ($D50_{15}$, T_{15} and F_{15}) in connection with other assumed parameters (W, T_{15}, M) must be investigated.

The equation presented in this study has the following advantages over Youd et al. [14] and Javadi et al. [20] equations:

i. The equation of Youd et al. [14] and Javadi et al. [20] are discontinued for free face ratio (1<W<5%).

ii. When both free face and ground slope are simultaneously present in the field, each pair of equations proposed by Youd et al. [14] and Javadi et al. [20] give two different values for D_h

iii. Under the Javadi et al. [20] equation, when the free face ratio (W) increases the value of lateral displacement increases and then decreases as shown in Fig.2. In other words, when the free face ratio increases beyond 30%, the displacement decreases with increasing free face ratio which is not compatible with the concept of the phenomenon and also findings of other researchers.

iv. If free face ratio (W) is remained constant at 6 degree and ground slope parameter (S) is increased from 1 to 17%, Javadi et al. [20] equation predicts that the lateral displacement will remain constant. This trend is not compatible with the physics of the phenomena.

v. The equation of Youd et al.[14]was not successful to estimate the lateral spreading for the new 41 cases (Fig.3.b).

vi. Precision of the developed equation is examined by plotting the measured versus predicted values of displacement for all the data as shown in Fig. 3.a. The same prediction for Youd's model is presented in Fig. 3.b. A comparison of these two Figures indicates that GP model performs very well, with R^2, RMSE and MAE of 0.89, 0.9 and 0.6 for training and 0.85, 0.7 and 0.5 for testing stages.

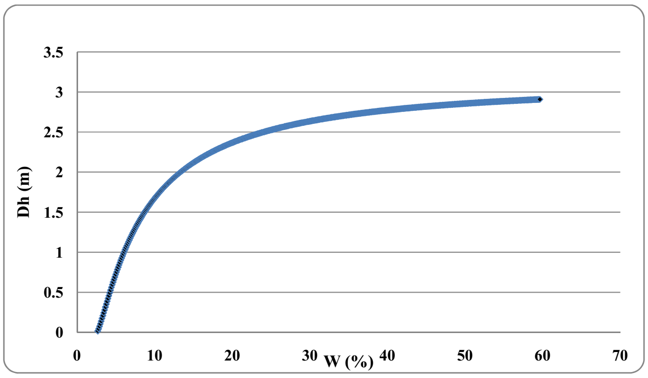
vii. According to the statistical parameters reported in 10, current equation is more accurate than any equation previously reported.

4. Summary and conclusions

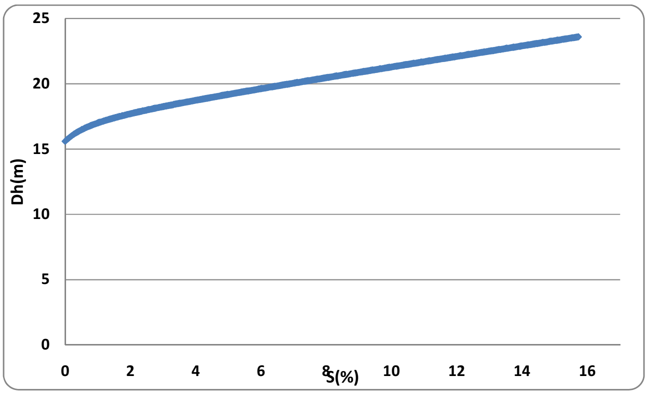
In this paper, neural network and genetic programming were used to predict the lateral spreading due to seismically induced liquefaction. A database including 525 case histories, for the first time, from 13 sites located in Japan, USA, Turkey and Taiwan was used to develop a new model.

Table 10 Statistical parameters of various models

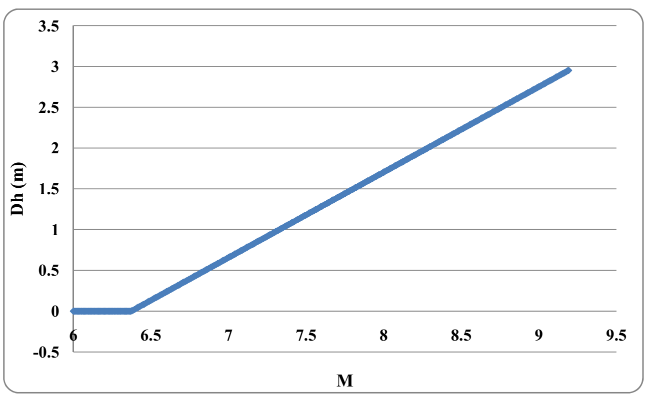
Model	Correlation Coefficient (R^2)%	Root-Mean-Square Error (RMSE)	Mean Absolute Error (MAE)
MLR-1999 (Bardet 6 parameters)	69	1.4	0.8
MLR-2002 (Youd et. al.)	71.2	1.4	0.7
GP-2006 (Javadi et al.)	81.6	1.3	0.7
GP-2013 (Current study)	88.6	0.8	0.6



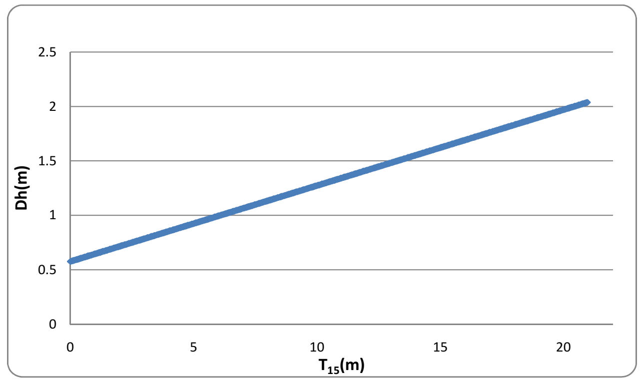
(a)



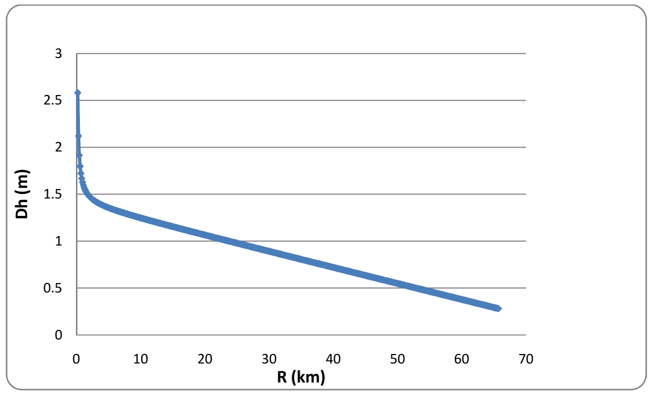
(b)



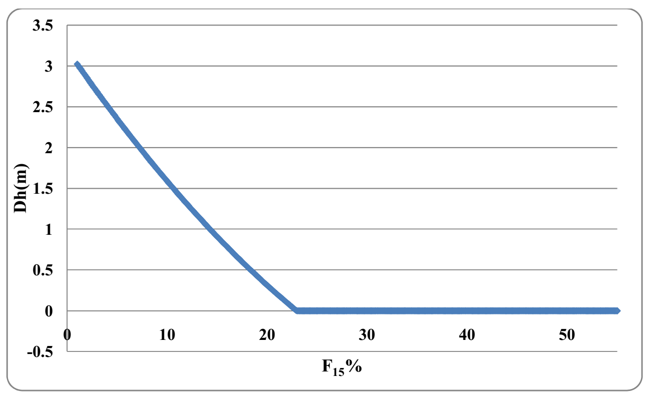
(c)



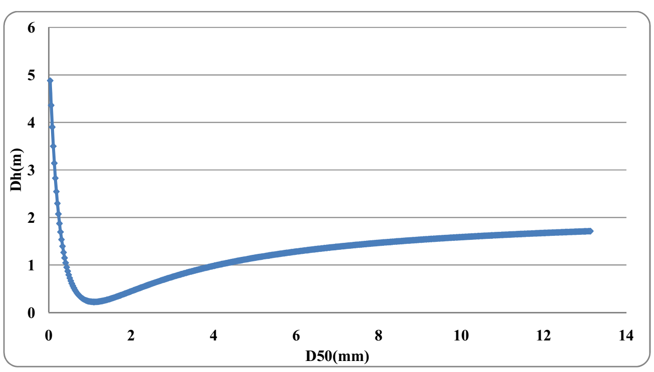
(d)



(e)



(f)



(g)

Fig. 1 Variation of the predicted horizontal displacement against different parameters when other parameters are set to their average, according to the proposed GP model

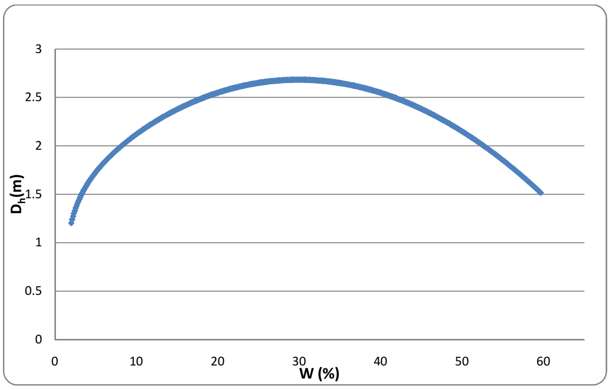
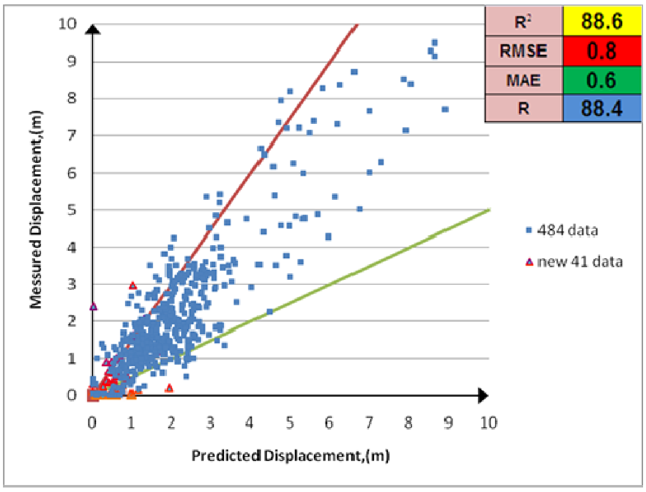
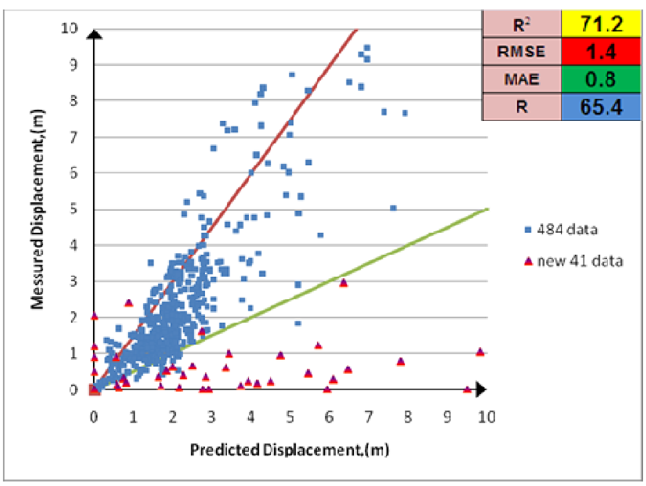


Fig. 2 Variation of the predicted horizontal displacement against W (%) according to Javadi et al. [20]



(a) Prediction of current (GP-2012) study for 525 data



(b) Prediction of Youd et al. [14] 's equation for 525 data

Fig. 3 Measured versus predicted lateral spreading

A sensitivity analysis was also carried out to study the relative importance of the factors, affecting lateral spreading. This sensitivity analysis indicated that the two factors of soil and earthquake have the most significant effect on the predicted displacements while the topography parameters have less impact on displacements. As a result of ANN analysis, seven parameters of M, R, T_{15}, F_{15}, $D50_{15}$, S and W for predicting lateral spreading were used as input parameters to develop the GP-based model. The proposed model, as indicated in Fig. 3, showed a reasonably good performance for all the data sets with (R^2 =88.6, RMSE=0.8 and MAE=0.6).

A parametric study was also performed to investigate the behavior of model for different conditions. The observed trends of the proposed model are in good agreement with the results reported by other researchers.This accuracy shows the superiority of the GP model over other traditional equations and suggests that the proposed equation can be applied for engineering practice.

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