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Presenting displacement-based nonlinear static analysis method to calculate structural response against progressive collapse

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Abstract

The current building codes provide limited prescriptive guidance on design for protection of buildings due to progressive collapse. Progressive collapse is a situation in which a localized failure in a structure, caused by an abnormal load, such as explosions or other happenings. Three procedures, often employed for determination of the structural response during progressive collapse i.e. linear static procedure (LSP), nonlinear static (NSP) and nonlinear dynamic (NDP) analyses. In nonlinear static analysis, a force-based method is applied and the structure is pushed down to the target force. In this research, a new displacement-based method will be proposed for nonlinear static analysis. In displacement-based method, the structure is pushed down to target displacement instead of target force (similar to the one in seismic pushover analysis). To make a nonlinear static analysis, instead of increasing the load around the area of the removed column, a maximum displacement is calculated and the upper node of the removed column is pushed up to target displacement. Here, to determine the target displacement, results from nonlinear dynamic and linear static analyses are compared. This paper tries to present a formula to calculate the target displacement using the linear static rather than the nonlinear dynamic analysis. For this reason, 3 buildings with 3, 5 and 10 stories have been seismically designed and studied. The results show that, this method is much more accurate in comparison to the recommended approach in current codes. Also, this method does not have the limitations of force-based nonlinear static analysis.

Keywords: *Progressive collapse, Linear static analysis, Nonlinear dynamic analysis, Nonlinear static analysis, Target force, Target displacement.*

1. Introduction

Progressive collapse is defined as the spread of an initial local failure from element to element, eventually, in the collapse of an entire structure or a disproportionately large part of it [1]. Based on old codes, designing of structures for progressive collapse is often done indirectly and with the help of special level of structural integrity [2]. In current codes to control the progressive collapse, an alternative method is preferred in many cases. In this method, the structure shall be able to bridge over vertical load-bearing elements that are notionally removed [3]. Three analysis procedures i.e. Linear static (LSP), Nonlinear static (NSP) and Nonlinear dynamic (NDP) are employed for such an alternative path. The nonlinear dynamic analysis is the most accurate one where geometric/material nonlinearity as well as dynamic effects are considered during a time history analysis. However, this approach is quite expensive because the sophisticated finite element modeling is required to account for all possible types of nonlinearities.

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With respect to two other given analysis procedures such as linear static and nonlinear static analysis, they achieve a quick response. These methods study the building loads in bays adjacent to the removed element and floors above it which are increased with a factor and structure adequacy. In order to increase the loads in the nonlinear and linear static methods, the DIF (Dynamic Increase Factor) and LIF (Load Increase Factor) are used, respectively. With regards to the nonlinear static force method and its dynamic increase factor, different researchers have come up with some formulas:

Analyzing a couple of steel frames subjected to column loss, Stevens [4] recommended the following formulas for LIF and DIF:

$$LIF = 0.765m + 1.235 \tag{1}$$

$$DIF = 1.44m^{-0.12} \tag{2}$$

In the first equation, m represents the total rotation divided by yield rotation, while it is equal to the plastic rotation divided by yield in the DIF expression [5].

Following the above procedure, Marchand [6] presented separate formulas for steel and concrete structures which presently are used to calculate the increase factor in UFC 4-023-03 [3].

The empirical formula for LIF factor is as follows [6]:

$$LIF = 0.9m + 1.1 \qquad for \quad steel \tag{3}$$

$$LIF = 1.2m + 0.8 \qquad for \quad RC \tag{4}$$

The empirical formula for DIF factor is as follows [6]:

$$DIF = 1.08 + \frac{0.76}{\frac{\theta_{pra}}{\theta_y} + 0.83} \qquad for \quad steel \tag{5}$$

$$DIF = 1.04 + \frac{0.45}{\frac{\theta_{pra}}{\theta_y} + 0.48} \qquad for \quad RC \tag{6}$$

Where, θ_y and θ_{pra} are the yield and the plastic rotation angle, respectively. According to these formulas, both the elastic LIF and DIF are equal to 2.0 as m=1 or $\theta_{pra}/\theta_y=0$.

Tsai [5] used another formula to calculate the above factors:

$$LIF = \frac{2\mu^2}{1+\alpha(\mu-1)^2 + 2(\mu-1)} \tag{7}$$

$$DIF = \frac{2\mu[1+\alpha(\mu-1)]}{1+\alpha(\mu-1)^2 + 2(\mu-1)} \tag{8}$$

Where, α is the post-elastic stiffness ratio of the model, μ represents the ductility demand and may be regarded as the displacement or rotation ductility [5].

Min Liu [7] innovated a dynamic increase factor for nonlinear static analysis w ith max (M_u/M_p) parameter, where the max operator is applied to all affected beams that are adjacent to and above the removed column, and M_u and M_p are the factored moment demands under original unamplified static gravity loads and the factored plastic moment capacity, respectively [7]. He presented the following equation for two kinds of columns (interior and exterior) on condition that the structure has enough residual capacity [7]:

For exterior column: if

$$\max\left(\frac{M_u}{M_p}\right) \le 0.5 \Rightarrow DIF = 1.15\max\left(\frac{M_u}{M_p}\right) + 1.12 \tag{9}$$

For interior column: if

$$\max\left(\frac{M_u}{M_p}\right) \ge 0.5 \Rightarrow DIF = 0.58\max\left(\frac{M_u}{M_p}\right) + 1.55 \tag{10}$$

Further, Liu proposed the following equation for both interior and exterior columns on conditions that the structure does not have enough residual capacity.

$$DIF = 0.84 + 1.23 / \left(2.95\max\left(\frac{M_u}{M_p}\right) - 0.28\right) \tag{11}$$

If one of the members of the main structure is destroyed suddenly, the rest of the members are able to bridge over other elements and have an alternative path to transfer the load [1, 2].

θ_{pra}/θ_y parameters have been used in most of these methods where θ_{pra} is the plastic rotation angle and is determined based on the acceptance criteria. In other words, the capacity of a member matching with ASCE 41-06 [8] or UFC 4-023-03 [3] codes, causes some inaccuracies because these parameters are indicative of the rotation capacity and in the structure we study the aforementioned member may never reach its capacity. Karimiyan et al. [9] and Shahrouzi et al. [10] applied progressive collapse for seismic design.

In this research, a new displacement-based method will be proposed for nonlinear static analysis. In displacement-based method, the structure is pushed down to target displacement instead of target force (similar to the one in seismic pushover analysis).

2. Recommended Method

As mentioned earlier, there are two major flaws in the methods presented for the nonlinear static analysis in alternative path method. The first problem is that these approaches are force-based which result in ignoring the post-elastic stiffness. The second defect is their using of θ_{pra}/θ_y parameter that indicates the structure capacity although the intended structure may never reach its capacity point in the progressive collapse phenomena.

With respect to limitations of the force-based nonlinear static method, the current study has tried to present a new nonlinear static displacement-based approach to control the progressive collapse. Accordingly, to make a nonlinear static analysis, instead of increasing the load around the area of the removed column, a maximum displacement is calculated and the upper node of the removed column is pushed downward as much as the target displacement (similar to the one in seismic pushover analysis). In case of the members' response to the column removal scenarios, it can be said that the building is resistant to the progressive collapse. Besides, to calculate the target displacement, the factored moment demand of the affected beam is used which shows the need of the structure, instead of using the parameters such as the plastic rotation angle which indicates the structure capacity alone. After modeling the structure and removing the column that is being studied through a linear static analysis under original unamplified static gravity loads (GL), this method suffices to calculate the maximum moment in affected beams that are adjacent to the removed column as well as the maximum displacement in the upper node of the removed column. At this stage, to calculate the M_R=max (M_u/M_p) parameter would probably be very easy with the help of plastic moment capacity ($M_p=Z*F_{ye}$) of the affected beams. Now, with the help of M_R parameter and the proposed formulas

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of the current research, the target displacement is computed to control the progressive collapse that had resulted from the removal of the column under study. At this point, through a nonlinear static analysis the upper joint of the intended column is stretched down as much as the target displacement without changing the loading or the geometry of the structure, and finally the structure responses are examined.

3. The Advantages of the Proposed Method

Compared to the force-based method, recommended by UFC 4-023-03 [3], the displacement-based method presented in this paper has the following advantages:

a) The accuracy of the displacement-based method is much remarkable.
b) Calculating the target displacement for each scenario is based on structural requirement, unlike the DIF factor calculation that is based on the structural capacity.
c) Lack of need to make changes in structure loading in this method that not only facilitates analysis but prevents the probable mistakes while loading.

4. How to Calculate the Target Displacement Formula

Actually, the target displacement is the maximum displacement resulting from the nonlinear dynamic analysis (Δ_{ND}) and hence, this paper has tried to present a formula to calculate the target displacement using the linear static rather than the nonlinear dynamic analysis, especially with regard to the position of the removed column as well as the number of beams within the affected bays immediately adjacent to the intended column. With the availability of the target displacement in removal scenario for each column in the alternative path method, the structural adequacy against the progressive collapse can be estimated through the nonlinear static analysis by pulling down the upper joint of the removed column as much as the target displacement. In this study, the target displacement has been defined as a function of M_R=max (M_u/M_p). M_u is the factored moment demand of the affected beam under original unamplified static gravity loads (Fig. 1) and M_p is the plastic moment capacity of an affected beam. $M_p= \Omega_0 Z F_y$, where Ω_0= overstrength factor, Z= cross-sectional plastic modulus, and F_y= steel yield stress. This parameter actually shows the level of nonlinearity of the sections while loading the building.

Original unamplified static gravity loads is as follow [2]:

$$GL = (0.9 or 1.2)D + (0.5L or 0.2S) \tag{12}$$

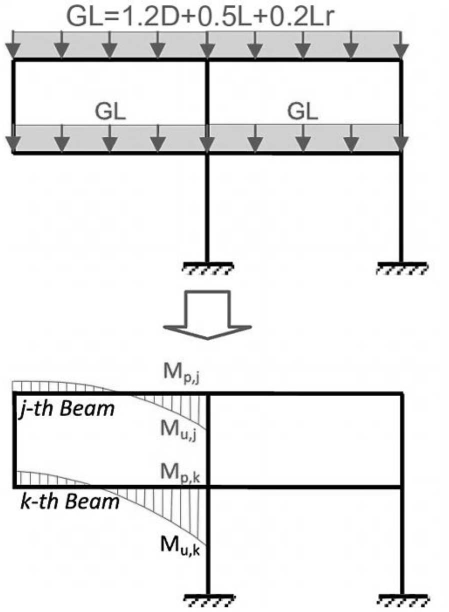

Fig. 1 Linear static analysis under original unamplified static gravity loads to calculate M_R=max(M_u/M_p) [7]

4.1. Step by step method to calculate the target displacement

The study, here, tries to elaborate on different steps to devise the target displacement formula for interior and exterior columns based on M_R parameter.

Step 1: A nonlinear dynamic analysis is performed to calculate the maximum displacement of the upper node of the removed column. For this, the structure is analyzed statically under the gravity load at first. Then according to Fig. 2, the reaction of the removed column is calculated and put in the next model (in opposite directions) that is an equilibrium one. The intended column is removed and so the reactions in less than one tenth of the period associated with the structural response mode for the vertical motion of the bays above the removed column through a time history analysis. The maximum displacement achieved from this analysis is called Δ_{ND}.

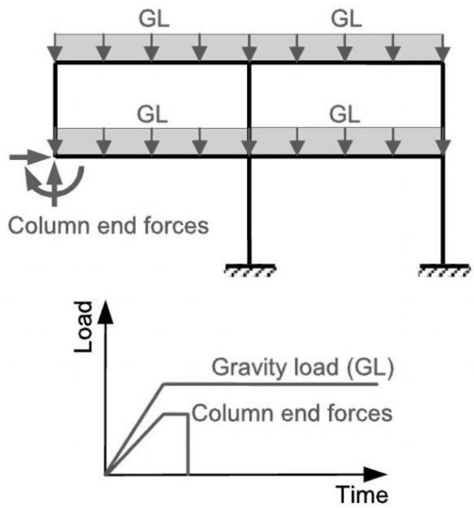

Fig. 2 Dynamic loading to calculate the maximum dynamic displacement [7]

Step 2: A linear static analysis under original unamplified static gravity loads is performed while the intended column has been removed. This leads to the maximum displacement of the upper node of the removed column (Δ_{LS}) and the maximum moment (M_u) in affected beams.

Step 3: $C=\Delta_{ND}/\Delta_{LS}$ versus $M_R=\max(M_u/M_p)$ of all different scenarios of column removal is drawn for all data points to devise the intended formula. The C is the conversion coefficient of the linear static displacement to the target displacement.

5. Modeling

This part highlights the steps taken to devise the target displacement formula for intermediate ductility steel moment frame structures. As such, 3 buildings with 3, 5 and 10 stories have been seismically designed and studied. The assumed height of story is 3.2 meters with span length of 6 meters. The gravity loads are the same for all three buildings i.e. dead loads of 6 KN/m^2, live loads of 2 KN/m^2 and 1.5 KN/m^2 for the snow on the roof. The structural design is based on the Iranian codes [11, 12] and AISC 360-05 [13].

The steel material used to analyze the progressive collapse in this survey has yield strength of 240 MPa and tensile strength of 370 MPa. An over-strength factor Ω_0 of 1.1 is used to convert steel strength from lower-bound to the expected values [8]. Member sizes for the three steel frames are listed in Table 1. A single section size is used for all beams at a given floor or the roof level. All stories columns have been studied.

Table 1 Member's dimensions of three steel building frames

BAY=6m	3 STORY		5 STORY		10 STORY	
	BEAM	COLUMN	BEAM	COLUMN	BEAM	COLUMN
1st story	IPE 360	BOX 35-1.2	IPE 450	BOX 45-1.5	IPE 550	BOX 55-2
2nd story	IPE 360	BOX 30-1	IPE 450	BOX 45-1.5	IPE 550	BOX 50-2
3rd story	IPE 300	BOX 25-1	IPE 450	BOX 40-1.2	IPE 550	BOX 50-2
4th story			IPE 400	BOX 35-1.2	IPE 550	BOX 45-2
5th story			IPE 300	BOX 30-1	IPE 550	BOX 45-2
6th story					IPE 550	BOX 45-1.5
7th story					IPE 450	BOX 45-1.5
8th story					IPE 450	BOX 40-1.2
9th story					IPE 400	BOX 40-1.2
10th story					IPE 360	BOX 35-1.2

To investigate and compute the load redistribution behavior when columns are removed in the progressive collapse, the SAP2000 [14] program has been utilized. Both nonlinear effects including geometric and materials are considered for this program application. Program defaults plastic hinges for columns, specifications of a-row of Table 5-5 from ASCE 41-06 [8] for beams are used. In this article, the removed columns are divided into: exterior (with 2 or 3 joined beams) and interior (with 4 joined beams). For each of these columns, a formula is presented to estimate the target displacement.

6. Results

Here, an elaboration is made on the detailed procedure that was taken to devise the target displacement of progressive collapse for an exterior column in a 3-story building with a 6-meter span. For the studied column, the nonlinear dynamic analysis was applied. Fig. 3 shows the displacement of the upper node of the removed column in the time history analysis.

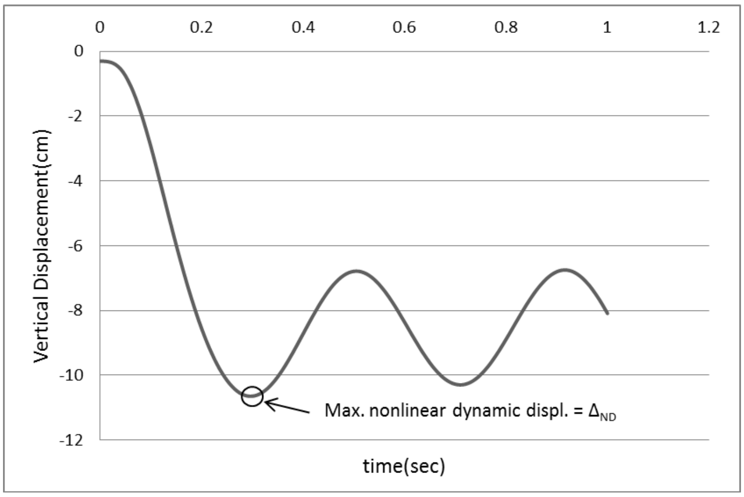

Fig. 3 Time history of vertical displacement at upper node of the suddenly removed column in non-linear dynamic analysis procedure

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For this column removal scenario, different loadings for building were used in order to consider their effects. The same steps were taken separately for each loading. During a linear static analysis, the maximum displacement resulting from the loading in return for original unamplified static gravity loads was calculated. Also the maximum moment demand was determined as mentioned in Table 2. As the load increased for the intended column, the $C=\Delta_{ND}/\Delta_{LS}$ diagram versus $M_R=\max(M_u/M_p)$, changed with an upward trend as shown in Fig. 4.

Table 2 Gained data from analyzing the examined building with gradual increasing loads

$C=\Delta_{ND}/\Delta_{LS}$	Δ_{LS}(cm)	M_u(ton-m)	$M_R=\max(M_u/M_p)$
2.021	3.995	29.16	1.084
2.167	4.383	32.02	1.190
2.385	4.771	34.88	1.297
2.706	5.159	37.74	1.403
3.167	5.547	40.6	1.509
3.833	5.935	43.47	1.616
4.285	6.13	44.9	1.669
M_p(IPE360)=26.9 ton-m			

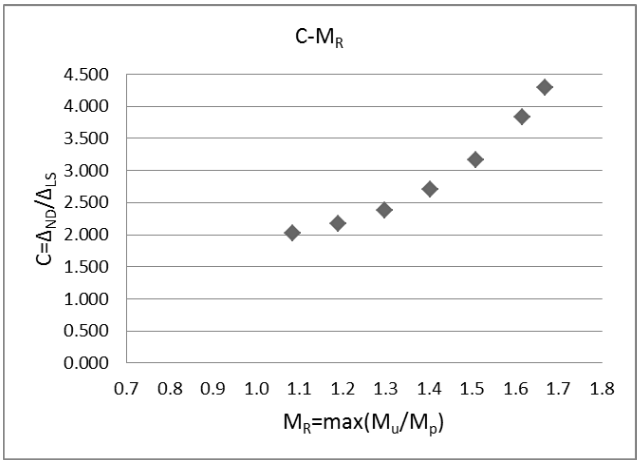

Fig. 4 $C=\Delta_{ND}/\Delta_{LS}$ versus $M_R=\max(M_u/M_p)$

The above operation (for all the scenarios) leads two diagrams being highlighted in Figs. 5 and 6. The interior and exterior columns are indicated with curves, for each of these diagrams Eq. (13) and (14) devised for the target displacement in the nonlinear static progressive collapse analysis. The curve fitting is carried out to empirically derive the following equation for internal column removal scenario:

$$C = 11.55 \times M_R^2 - 22.61 \times M_R + 13.13 \quad (13)$$

To calculate the C factor for external columns Eq. (14) is derived as:

$$C = 7.27 \times M_R^2 - 15.88 \times M_R + 10.7 \quad (14)$$

To compute the target displacement of each scenario, there is a need to put the amount of C in the following equation:

$$\Delta_{target} = C \times \Delta_{LS} \quad (15)$$

As known, for an un-damped structure in the linear elastic manner, to calculate static displacement under dynamic loading, amplification factor equals 2 [15]. In other words, the dynamic displacement resulting from a constant load which is suddenly put on a structure, is twice as much as the displacement of the same load when it is put on the building slowly and statically. Regarding Figs. 5 and 6, for exterior columns in return for $\max(M_u/M_p) \leq 1.0$ the damaged frame has enough residual capacity to remain in the elastic manner and the C factor equals 2. Also, for interior columns in return for $\max(M_u/M_p) \leq 0.9$ the damaged frame has enough residual capacity to remain elastic and the C factor equals 2. When $\max(M_u/M_p)$ exceeds 1.0, the remaining capacity of exterior columns is not enough to remain in the elastic manner and it is expected that the structure enters higher level on nonlinearity. Concerning interior columns, this limitation decreases to 0.9 and such a difference can be the result of a higher level of geometrical nonlinearity and/or material nonlinearity in exterior column removal scenarios. It should not be forgotten that the application of number 1.0 for exterior columns and 0.9 for interior columns is just to ease calculations and it is reasonable to expect that such a difference is small and may well be neglected from a practical point of view. We should also bear in mind that the target displacement method is an accurate one and does not have flaws and limitations of UFC 4-023-03 [3] (load increase factor).

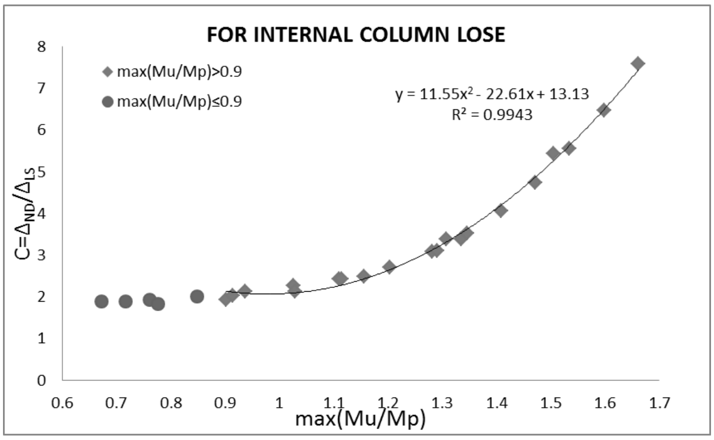

Fig. 5 The C coefficient as a function of M_R for internal column

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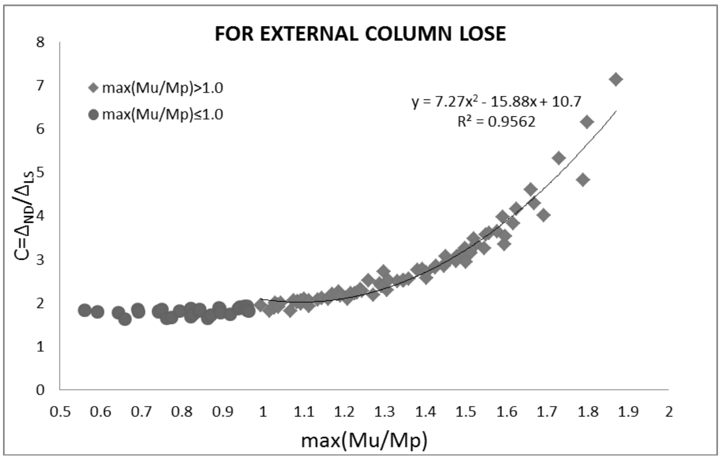

Fig. 6 The C coefficient as a function of M_R for external column

7. How to Use the Target Displacement Formula

After devising the target displacement formula, the following steps are taken to control the progressive collapse in buildings:

Step 1: Take a linear static analysis under original unamplified static gravity loads while the intended column is removed and calculate the maximum displacement of the upper node of the removed column (Δ_{LS}) and the maximum moment (M_u) in the affected beams resulting from these loads.

Step 2: Calculate the C factor using a special formula regarding the position of the column which is supposed to be removed.

Step 3: The target displacement can be calculated through multiplying the C factor with the linear static displacement (Δ_{LS}) gained from the step one.

Step 4: The upper node of the removed column is pulled down through a nonlinear static analysis as much as the target displacement. If plastic hinges in structure do not exceed the acceptance criteria defined in ASCE 41-06 [8] or UFC 4-023-03 [3] the structure will have adequate capacity to prevent the progressive collapse resulting from the removal of the intended column.

8. Case Studies

The paper compares the accuracy of the proposed method and calculates the target displacement of a 3-story building with non-similar spans. Later on, the target displacement is also compared with the maximum dynamic displacement and the maximum displacement obtained from the presented method in UFC 4-023-03 [3]. Table 3 indicates the member sizes of the structure. Fig. 7 highlights the story height 3.2 meters and the distance between columns. It should be noted that the proposed building is completely different from the ones used to devise the target displacement formula.

For this building, three columns i.e. the corner column (CC), the penultimate column (PC) and the internal column (IC) have been removed which are shown in Fig. 8.

Table 3 Dimension of members of three story steel frame with non-similar spans

Non-similar spans	3 STORY	
	BEAM	COLUMN
1st story	IPE 360	BOX 35-1.2
2nd story	IPE 360	BOX 30-1
3rd story	IPE 300	BOX 25-1

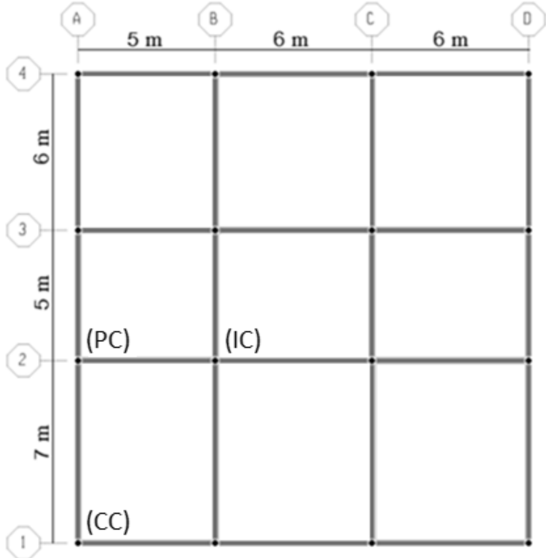

Fig. 7 Plan of a 3-story frame with non-similar spans

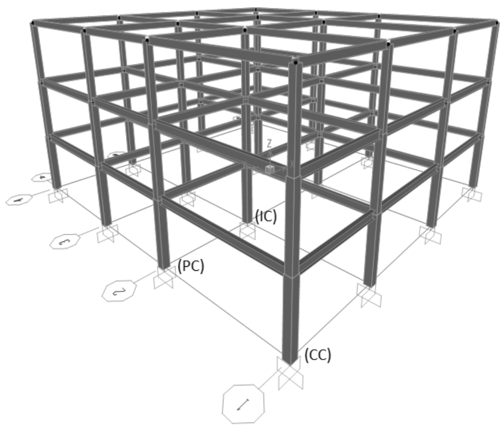

Fig. 8 the 3D view of the building and to be removed columns

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Table 4 shows the steps taken to calculate the target displacement for each of the columns in this structure under dead load of 7.5 KN/m^2, live load of 3.5 KN/m^2 and snow load of 2 KN/m^2. At the end, the acquired target displacement is compared with the amounts resulting from nonlinear dynamic and nonlinear static procedures, each of which calculated separately based on the necessities of the UFC 4-023-03[3]. This indicates that the proposed method is more accurate than the one presented in UFC 4-023-03[3]. All moment connections are improved WUF with Bolted Web.

Table 4 The obtained results from examining progressive collapse in studied structure

	Corner Column (CC)	Penultimate Column (PC)	Internal Column (IC)
Max (M_u) (ton-m)	38.16	31.89	38.16
Δ_{LS} (cm)	5.03	3.8	4.85
M_p (ton-m)	26.9	26.9	26.9
Max (M_u/M_p)	1.380	1.186	1.418
C	2.63	2.1	4.29
Δ_{target} (cm)	13.22	7.98	20.8
Δ_{ND} (cm)	12.86	8.652	18.45
Δ_{NS} (cm)	9.39	6.133	24.1
Error rate of NSP method of UFC 4-023-03 %	-26.98	-29.11	30.62
Error rate of proposed method	2.80	-7.76	12.73

The information given in the table is as follows:

Δ_{ND}: the maximum displacement resulting from the nonlinear dynamic analysis

Δ_{NS}: the maximum displacement resulting from the nonlinear static analysis under load combination along with dynamic increase factor (DIF*GL)

Δ_{LS}: the displacement resulting from the linear static analysis under original unamplified static gravity loads (GL).

C: the proportion of the maximum dynamic displacement to the maximum linear static displacement which is calculated based on the target displacement formula introduced in this article.

As an example, the process of calculating the target displacement for the corner column is explained. First, the maximum moment of adjacent beams of removed columns called M_u is calculated through a linear static analysis under original unamplified static gravity loads (GL). Then through dividing M_u by M_p, and putting the result in the exterior column equation Eq. (14) the amount of C is obtained. At this stage, to calculate the target displacement, we just need to multiply the amount of C by the upper node vertical displacement of the removed column resulting from linear static analysis (Δ_{LS}).

Table 4 compares the results of the proposed method and NSP method relative to nonlinear dynamic analysis. For instance, the error rate of NSP method ($\frac{\Delta_{ND}-\Delta_{NS}}{\Delta_{ND}} = 26.98\%$) compared to new method ($\frac{\Delta_{ND}-\Delta_{TARGET}}{\Delta_{ND}} = 2.8\%$) is equal 26.98% and 2.8% respectively.

9. Conclusion

So far, all analytical approaches to alternative path method have been force-based. The displacement-based approach for nonlinear static analysis is an alternative path to the UFC 4-023-03 dynamic increase factor force method. Based on analyses which were performed for the mentioned buildings, an empirical formula has been devised to calculate the maximum vertical displacement of the removed column's upper node, which is the target displacement.

A comparison of results from the proposed nonlinear static method and UFC 4-023-03 [3] nonlinear static method indicates that the new method is more accurate than the existing ones. The other advantages that can be mentioned are its ability to distinguish interior and exterior columns and present a separate formula to calculate the target displacement for each of the columns that leads to more accuracy.

Another strong point of this approach is the use of the max(M_u/M_p) parameter instead of θ_{pra}/θ_y which has been applied in nonlinear static procedure in UFC 4-023-03 [3]. In other words, while θ_{pra}/θ_y parameter indicates the capacity of the structure; the max(M_u/M_p) parameter shows the structural demand.

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