# Evaluation of conventional methods in Analysis and Design of Railway Track System

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***Abstract:*** *Current practices in railway track analysis and design are reviewed and discussed in this paper. The mechanical behavior of railway track structure comprising of various components has not been fully understood due to the railway track structural complexity. Although there have been some improvements in the accuracy of current track design methods in recent decades, there are still considerable uncertainties concerning the accuracy and reliability of the current methods. This indicates a need for a thorough review and discussion on the current practices in the analysis and design of railway track systems. In this paper, railway design approaches proposed by various standards along with the results of a wide range of technical researches are studied and necessary suggestions are made for the improvement of current practices in the analysis and design of railway tracks.*

***Keywords:*** *railway track, track components, analytical models, design criteria, track design procedure.*

## 1. Introduction

Ballasted railway tracks mainly comprise two main parts: superstructure and substructure. Steel rails, various types of rail fasteners, timber, steel, or concrete sleepers, and granular ballast, sub-ballast, and subgrade materials are major components used in railway track construction. Historically, the understanding of track structural behavior has been facing difficulties. This is due to different mechanical properties of track components from one side, and complex interaction between track components from the other side.

Wide range of track configurations could be designed and constructed. This makes railway track structure to be subjected to regular changes. Consequently, based on theoretical and experimental investigations, large number and sometimes contradictory design criteria have been suggested by railway authorities and practitioners. Such a diversity of design criteria usually makes railway track design procedure a difficult task. This highlights the necessity of a need for a thorough review of the currently used railway track analysis and design methods.

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This paper focuses on a wide range of railway related design codes as well as a great deal of recent technical researches the suggestions of which are usually utilized in the analysis and design of railway tracks. Results of this review is presented and discussed in order to indicate the required improvements needed to be considered in the current railway track analysis and design.

## 2. Conventional Methods of Analysis and Design of Railway Tracks

Generally, railway track systems are designed to provide a smooth and safe running surface for passing trains. They also serve to sustain the loads imposed to track structure mainly as a result of trains passages and temperature changes.

Due to great variety of structural elements used in track system, it is usually more practical to perform the analysis and design procedure for each element as a single structural unit. Such an approach, subsequently, includes the interaction between track components through the definition of suitable boundary conditions and load transfer patterns. Furthermore, since the dynamic response characteristics of the track are not sufficiently well understood to form the basis of a rational design method, current practices greatly relies on relating the observed dynamic response to an equivalent static response. This is carried out by making use of various load factors.

This method is being widely accepted and used in analysis and design of railway track systems.

The track is designed utilizing load bearing approach [1] to ensure that the concentrated loads of the wheels are transferred to the formation while ensuring that the strength of the components are not exceeded. Several important criteria are defined to secure this objective. These criteria mainly include limits on rail and rail fastener stresses and deflections, sleeper strength, pressure distributed in sleeper-ballast interface, and the stresses distributed on granular supporting layers underneath the track.

### 2.1. Rail

Rail as the most important track element subjected to wheel loads must be able to securely sustain these loads applied in vertical, lateral, and longitudinal directions and subsequently transfer them to the underlying supports. Rail is the track element which is in direct contact with the rolling stock. It is therefore very necessary, in particular from a safety point of view, to ensure the proper functioning of rails in the track system.

The most important recommended criteria used in the conventional rail analysis and design procedure are shown in Figure 1. As it is illustrated in this figure, rail design criteria are mainly divided into two categories. Structural strength criteria include wheel-rail contact stresses and rail bending stresses. Having satisfied the structural strength criteria, the serviceability requirements should be completely met for a specific rail section to ensure its proper structural and operational performance. In fact, aside from the calculation and controlling of rail

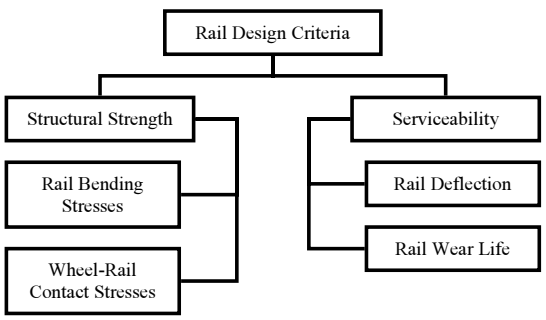



**Fig. 1.** Recommended rail design criteria

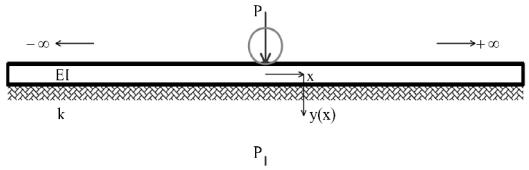



**Fig. 2.** Beam on elastic foundation model

stresses, it is critical that the design engineer has deep understanding about the real operating conditions the railway track may experience.

Current practices in the calculation of rail bending moments and vertical deflection are mainly based on "beam on elastic foundation" model. This model proposed for the first time by Winkler in 1867 and thereafter developed by Zimmerman in 1888 [2]. The basic assumption in the Winkler model is that the deflection of the rail at any point is proportional to the supporting pressure under the rail (see Figure 2).

The corresponding equations for the calculation of rail bending moment and rail deflection are as follow:

$$y(x)=\frac{P\beta e^{-\beta x}}{2u}(\cos\beta x+\sin\beta x) \quad (1)$$

$$M(x)=\frac{P}{4\beta}e^{-\beta x}(\cos\beta x-\sin\beta x) \quad (2)$$

where, y(x) and M(x) are the vertical deflection and the bending moment of the rail at the distance "x" from the load point, respectively. The parameter $\beta$ is defined with the following equation:

$$\beta=\left(\frac{u}{4EI}\right)^{0.25} \quad (3)$$

Winkler model is basically developed for a continuosly supported beam on an elastic foundation. This approach neglects some real conditions of railway tracks. First, the assumption of continous support under the rail does not reflect the effects of actual discrete support provided by cross sleepers. Second, this model does not include the interaction between support materials (i.e. ballast, sub-ballast, and subgrade materials) and simply uses a Bernoulli-Euler beam theory to calculate rail deflections and bending moments. Moreover, different track supporting layers are not clearly distinguished,

and track support is considered as a one-layer component. Finally, it is assumed that supporting sleepers fastened tightly to the rail would resist against rail bending through their rotational stiffness. The latter is another area of deficiency in Winkler model.

Some researchers have questioned the reliability of the Winkler model. As a result, more realistic approaches are proposed and some improvements are made in the basic "beam on elastic foundation" model. For instance, "beam on discrete support" model has been developed and more recently analyzed using a practical energy approach [3] to compensate for the errors caused by the assumption of continuous support under the rail. "Pasternak foundation" and "double beam" models are also introduced [2] to take into account the interaction between track supporting layers and multi-layer nature of track support, respectively. Kerr [1] has reported the result of a research in which the effect of the rotational stiffness of the sleepers in calculation of rail deflection and bending moment are considered.

Design procedure for a specific rail section always starts with the calculation of the design wheel load. This load is defined as the product of static wheel load and a corrective factor known as dynamic impact factor to compensate for dynamic as well as impact effects of wheel load resulted from wheel and rail surface irregularities. Taking into account various parameters which affect the magnitude of dynamic impact factor, several researchers along with railway authorities has been proposed different relationships for the estimation of this parameter. Some of these equations are summarized in Table 1.

The magnitude of vertical rail deflection calculated using Equation (1) is greatly dependent upon track modulus. Track modulus is defined as the load required producing unit vertical deflection in unit length of the rail. For typical tracks with light to medium rails, AREMA [5] recommends a value of 13.8 MPa when calculating vertical rail deflection.

The rail bending stress is usually calculated at the center of the rail base assuming the pure bending conditions to be applicable. The bending

**Table 1.** Recommended relationships for the calculation of dynamic impact factor [4]

| Developer | Equation |
|---|---|
| AREMA | $\varphi = 1+5.21\frac{V}{D}$ |
| Eisenmann | $\varphi = 1+\delta\eta.t$ |
| ORE | $\varphi = 1+\alpha'+\beta'+\gamma'$ |
| BR | $\varphi = \frac{8.784(\alpha_1+\alpha_2)V}{P_s}\left[\frac{D_j P_u}{g}\right]^{1/2}$ |
| India | $\varphi = 1+\frac{V}{58.14u^{0.5}}$ |
| South Africa | $\varphi = 1+4.92\frac{V}{D}$ |
| Clarke | $\varphi = 1+\frac{19.65V}{D u^{1/2}}$ |
| WMMTA | $\varphi = (1+3086*10^{-5}V^2)^{0.67}$ |
| Sadeghi | $\varphi = 1.098+8\times10^{-4}V+10^{-6}V^2$ |

stress at the lower edge of the rail head also may be critical if the vehicles impose high guiding forces between wheel flange and rail head while passing around the curves. Having calculated the magnitude of rail bending stress, comparison should be made between this stress and the allowable limit. AREMA [5] has recommended a practical methodology for calculation and controlling of rail bending stress based upon fatigue consideration and through the determination of several safety factors. According to this method, the allowable bending stress is defined as:

$$\sigma_{all} = \frac{\sigma_y - \sigma_t}{(1+A)(1+B)(1+C)(1+D)} \quad (4)$$

where, $\sigma_y$ is the yield stress of rail steel and $\sigma_t$ is the longitudinal stress due to temperature changes and can be calculated using the following equation:

$$\sigma_t = E.\alpha.\Delta t \quad (5)$$

The parameters A, B, C, and D in Equation (4) are safety factors to account for rail lateral bending, track condition, rail wear and corrosion, and unbalanced superelevation of track, respectively. Some of the recommended values of the above safety factors are presented in Table 2.

Wheel-rail contact stresses mainly include rolling and shear stresses. The magnitude of these stresses is greatly dependent upon the geometry

**Table 2.** Values of rail bending stress safety factors [6]

| Safety Factor | Researcher | | |
|---|---|---|---|
| | Hay | Clarke | Magee |
| A | 15% | 15% | 20% |
| B | 25% | 25% | 25% |
| C | 10% | 10% | 35% |
| D | 15%-20% | 25% | 15% |

of ellipsoidal wheel-rail contact patch. Many investigations have been carried out to develop reliable formulations for the calculation of these stresses. The most applicable formulas are those suggested by Eisenmann [7]. He conducted an analysis of rolling and shear stress levels in which the simplifying assumption of uniform distribution over the wheel-rail contact area was made. In this analysis wheel and rail profiles were also represented by a cylinder and a plane, respectively (see Figure 3).

Based on Hertz's theory, Eisenmann [7] suggested the following formula for the calculation of the mean value of the rolling contact stress:

$$\sigma_{mean}=\frac{P_s\times10^3}{2a\times2b} \tag{6}$$

where, 2b (mm) is the breadth of wheel-rail contact area. Eisenmann [7] adopted the value of 2b=12 mm. The contact length (2a) is also calculated from the following formula:

$$2a = 3.04\times\left[\frac{P_s.R_w\times10^3}{2b.E}\right]^{0.5} \tag{7}$$

The values of wheel loads transferred to the rail head through the contact area often exceed the yield limit of the contacting materials. In this situation, the resulting surface plastic deformations jointed with wear processes acts to

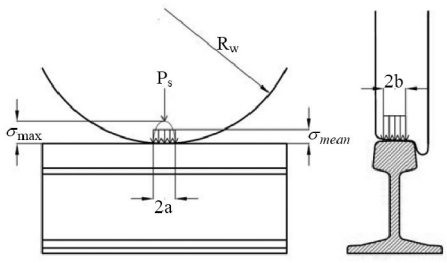



**Fig. 3.** Uniform distribution of wheel-rail contact stress [2]

flatten out the contact area. Therefore, the contact surface can be approximated by a rectangle of the length of 2a and the breadth of w based upon the assumption of contact between a plane (rail) and a cylinder (wheel). For such a condition, Smith and Liu [8] suggested the following formula for the calculation of the contact length:

$$2a = 3.19\times\left[\frac{P_s.(1-\nu^2).R_w\times10^3}{w.E}\right]^{0.5} \tag{8}$$

Considering required fatigue strength for rail steel, Eisenmann [7] proposed the limit value for mean rolling contact stress as a percentage of the ultimate tensile strength of rail steel. Based on this assumption, subsequent criterion is suggested:

$$\sigma_{all(roll)}=0.5\sigma_{ult} \tag{9}$$

Shear stress distribution is chiefly occurs in the rail head area and is in a close relationship with the magnitudes of normal principal stresses. Eisenmann [7] observed that the values of major and minor stresses do not follow the same reduction patterns with increasing depth from the rail head surface. Such a discrepancy results in the appearance of a maximum value of shear stress at a depth corresponding to half of the contact length. Maximum shear stress value is simply interrelated to mean rolling contact stress values and is given with the following equation:

$$\tau_{max} = 0.3\sigma_{mean} \rightarrow \tau_{max} = 410\sqrt{\frac{P_s}{R_w}} \tag{10}$$

As indicated earlier, the magnitudes of shear and rolling contact stresses are interrelated. Using the theory of shear strain energy applied for the condition in which the two principal stresses are compressive, the subsequent criterion for the shear stress limit could be obtained:

$$\tau_{all} = \frac{1}{\sqrt{3}}\sigma_{mean} \rightarrow \tau_{all} = 0.3\sigma_{ult} \tag{11}$$

Criteria related to the performance of the rails under operating conditions mainly include rail

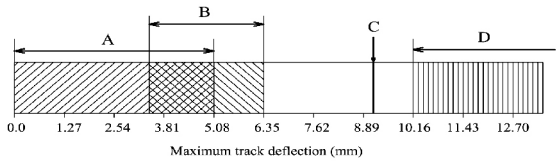



**Fig. 4.** Track deflection criteria for durability [9]

vertical deflection and rail wear life. These criteria are presented hereunder.

AREMA [5] has proposed a limiting range for the magnitudes of vertical rail deflections. According to this recommendation, extreme vertical rail deflections should be kept within the range of 3.175 to 6.35 millimeters. Lundgren and his colleagues [9] has incorporated this recommendation and proposed the diagram presented in Figure 4 as the limit values of vertical rail deflection. This diagram is based upon the capability of the track to carry out its design task.

Domains indicated in Figure 4 are described as follows:

A: Deflection range for track which will last indefinitely.
B: Normal maximum desirable deflection for heavy track to give requisite combination of flexibility and stiffness.
C: Limit of desirable deflection for track of light construction (with rails weigh < 50 kg/m)
D: Weak or poorly maintained track which will deteriorate quickly.

It should also be noted that values of deflection in Figure 4 do not include any looseness or play between rail and pad or pad and sleeper. In addition, these values represent deflections directly under the wheel load.

The other serviceability criterion is the rail wear life. Although many investigations have been carried out to develop a rational method for estimation of this parameter, the results at best are still empirical and have no theoretical support.

The University of Illinois has conducted a research on some U.S. railway tracks in order to investigate the rail wear rate and develop a method for the estimation of rail wear life. The following formula is suggested by the researchers of this university for the estimation of annual abrasive rail head area wear (mm²/year) [10]:

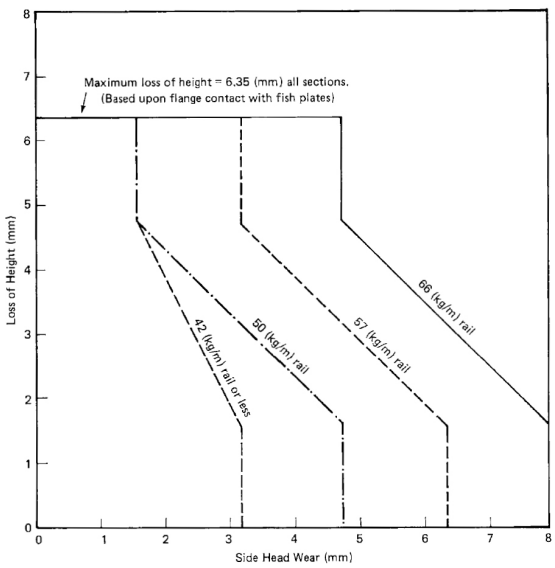
$$W_a = W_t.(1+K_w.D_c).D_A \tag{12}$$

Having estimated $W_a$ and considering the maximum rail head limit ($\theta_A$), the rail wear life could be calculated from the following formula:

$$T_y = \frac{\theta_A}{W_a} \tag{13}$$

Danzig and his colleagues [11] from the AREMA association also carried out extensive investigations to find a proper formulation for the estimation of rail wear life. Based on the results obtained, they suggested the following equation which represents the rail wear life in terms of MGT passed the track over a specific period of time:

$$T = \frac{1.839 K_C . K_G . K_R . R_{Wt} . (1.102 D_A)^{1.565}}{\sum_i^n \left[ \frac{1.102 D_i}{K_{V_i} . K_{A_i} . K_{S_i}} \right]} \tag{14}$$




Note: Values used in drawing the envelope of rail wear limits are:

| Loss of Height (mm) | Side Head Wear (mm) | | | |
|---|---|---|---|---|
| | 66 (kg/m) | 57 (kg/m) | 50 (kg/m) | ≤42 (kg/m) |
| 1.57 | 7.92 | 6.35 | 4.75 | 3.18 |
| 3.18 | 6.35 | 4.75 | 3.18 | 2.39 |
| 4.75 | 4.75 | 3.18 | 1.57 | 1.57 |

**Fig. 5.** Envelope of rail wear limits for loss of rail head hight and width [17]

Allowable rail wear limits have been proposed by many researchers and almost each railway authority has established its own specific criteria. For example, an envelope of maximum rail wear values for different rail sections is proposed by Canadian National Railway as presented in Figure 5. Using the diagrams outlined in this figure, maximum allowable rail head height and width loss could be determined.

The values indicate acceptable rail wear life usually range from 20 to 50 percent in rail head area reduction. The weight of unit length of the rail, the amount of MGT passed over the track during its service life, and the train speed are the most important parameters which determine the proper values of allowable rail wear limits to be chosen. The more weight of unit length of the rail, the greater amount of rail head area reduction would be allowed. Oppositely, the greater amount of MGT and higher values of train speed call for more limited rail head area reduction.

### 2.2. Sleeper

The sleepers play important roles in railway track system. The primary function of the sleepers is to transfer the vertical, lateral and longitudinal rail seat loads to the ballast, sub-ballast and subgrade layers. They also serve to maintain the track gauge and alignment by providing a stable support for the rail fasteners.

The vertical loads induce bending moments to the sleeper which are dependent upon the degree and quality of ballast layer compaction underneath the sleeper. Besides, the performance of a sleeper to withstand lateral and longitudinal loading is relied on the sleeper's size, shape, surface geometry, weight, and spacing [17].

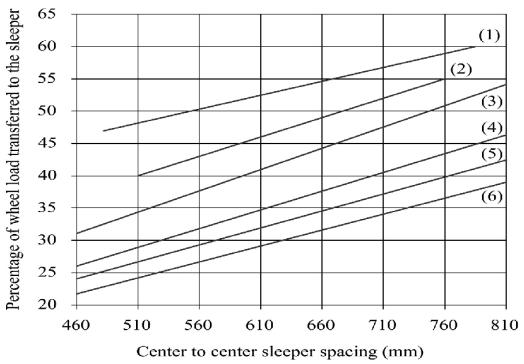
Current practices regarding the analysis and design of sleepers in vertical direction comprise three steps. These are [4]: 1) estimation of vertical rail seat load, 2) assuming a stress distribution pattern under the sleeper, and 3) applying vertical static equilibrium to a structural model of the sleeper.

Vertical wheel load is transferred through the rail and distributed on certain numbers of sleepers due to rail continuity. This is usually referred to as vertical rail seat load. The exact magnitude of the load applied to each rail seat depends upon several parameters including the rail weight, the sleeper spacing, the track modulus per rail, the amount of play between the rail and sleeper, and the amount of play between the sleeper and ballast [4]. Based on these considerations, various relations are proposed and summarized in Table 3.

However, for the purpose of simplification, it would be more practical to consider only the effect of some of the above mentioned parameters and define, for example, the value of vertical rail seat load as a function of sleepers' type and spacing. This approach is widely accepted by many railway authorities. For an

**Table 3.** Relations for the calculation of rail seat load

| Developer | Formula |
|---|---|
| Talbot [4] | $q_r = S.u.y_{max}.F_1$ |
| ORE [4] | $q_r = \bar{\varepsilon}.C_1.P$ * |
| UIC [13] (Concrete Sleepers) | $q_r = \frac{P_s}{2}(1+\gamma_p \times \gamma_v) \times \gamma_d \times \gamma_r$ |
| Austraila [14] (Concrete Sleepers) | $q_r = j \times P_s \times \frac{D.F.}{100}$ |
| Austraila [15] (Steel Sleepers) | $q_r = 0.5 \times F_2 \times P \times S \times \beta$ |
| Sadeghi [16] | $q_r = 0.474P \times (1.27S + 0.238)$ |
| * $\bar{\varepsilon}$ is defined as the ratio of $\bar{q}_r/\bar{P}_s$ in which $\bar{q}_r$ and $\bar{P}_s$ are mean values of rail seat load and static wheel load, respectively. $C_1$ is a coefficient usually equals to 1.35. | |




(1) Timber Sleeper
(2) Concrete Sleeper
(3) Concrete Sleeper (u = 55 N/mm2)
(4) Hardwood Timber Sleeper (u – 28 N/mm2)
(5) Steel Sleeper (u – 21 N/mm2)
(6) Softwood Timber Sleeper (u = 21 N/mm2)

**Fig. 6.** Diagrams for the estimation of rail seat load [12]

instance and as illustrated in Figure 6, AREMA [12] recommends diagrams in which the percentage of wheel load transferred to the sleeper are drawn against the sleeper spacing. The effect of sleeper type and track modulus is also included in these diagrams.

The exact contact pressure distribution between the sleeper and the ballast and its variation with time, will be of importance in the structural design of sleepers. When track is freshly tamped the contact area between the sleeper and the ballast occurs below each rail seat. After the tracks have been in service the contact pressure distribution between the sleeper and the ballast tends towards a uniform pressure distribution [4]. This condition is associated with a gap between the sleeper and the ballast surface below the rail seat. The most accepted contact pressure distribution patterns between sleeper and ballast are presented in Table 4.

Although some researches have been conducted to determine the in-track distribution of the pressure under the sleeper for design purposes such as those recommended by Sadeghi [16] (see Table 5), but the results are still not as practical as it is needed.

As indicated in Table 6 and to take into account the sleeper support condition as real as it is possible, it is usually presumed that the uniform pressure under the sleeper distribute in certain portion of the sleeper length (area). This length (area) is referred to as "Effective Length (Area)" and commonly shown with "L ($A_e$)" in the literature. This assumption is made to facilitate the procedure of design calculations. The static equilibrium in vertical direction is then applied to acquire the magnitude of contact pressure under the sleeper. A factor of safety is also included to account for variations in the sleeper support. Therefore, the average contact pressure between the sleeper and the ballast $P_a$ (kPa) can be acquired by:

$$P_a = \left(\frac{q_r}{B.L}\right)F_3 \qquad (15)$$

**Table 4.** Some contact pressure distribution patterns [4]

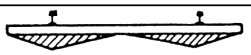
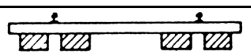
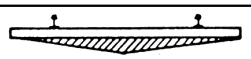
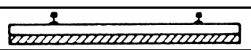
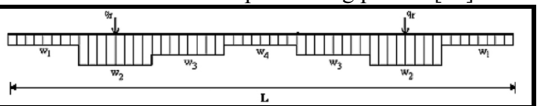
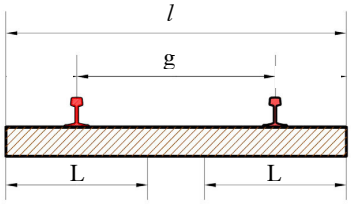
| Pressure Distribution | Remarks |
|---|---|
|  | Laboratory test |
|  | Principal bearing on rails |
|  | Tamped either side of rail |
|  | Maximum intensity in middle |
|  | Uniform pressure |

**Table 5.** In-track sleeper loading pattern [16]




| Pressure distribution pattern beneath sleeper | |
|---|---|
| After Tamping | $w_1 = 1.267\,q_r / L$ |
| | $w_2 = 2.957\,q_r / L$ |
| | $w_3 = 1.967\,q_r / L$ |
| | $w_4 = 1.447\,q_r / L$ |
| After Accumulative Loading | $w_1 = 1.596\,q_r / L$ |
| | $w_2 = 2.436\,q_r / L$ |
| | $w_3 = 1.974\,q_r / L$ |
| | $w_4 = 1.687\,q_r / L$ |

**Table 6.** Effective length (area) of sleeper support at rail seat




| Developer | Description |
|---|---|
| AREMA [12] | $A_e$: Two third of sleeper area at its bottom surface |
| UIC [13] | $A_e$ = 6000 cm$^2$ for $l$ = 2.5 m<br>$A_e$ = 7000 cm$^2$ for $l$ = 2.6 m |
| Australia [14, 15] | $L=(l-g)$ (1)<br>$L=0.9\times(l-g)$ (2) |
| Schramm [11] | $L=\frac{l-g}{2}$ |
| Clarke [11] | $L=(l-g)\left(1-\frac{(l-g)}{125t^{0.75}}\right)$ (3) |
| Clarke (simplified) [11] | $L=\frac{l}{3}$ |

(1) For bending moment calculation at rail seat
(2) For bending moment calculation at sleeper center
(3) the parameter "t" is the sleeper height

Having determined the sleeper loading pattern, its structural model could be defined for the calculation of sleeper bending moments. This subject has drawn the special attention of many railway researchers and consequently, several formulations have been suggested in the literature. All of these proposed formulas and methods could be classified based on sleeper type as shown in Figure 7. Formulations of these methods are also summarized in Table 7.

It should be noted that there are some differences between the sleeper structural models (i.e. sleeper loading patterns) used for the calculation of bending moments in each type of the sleepers indicated in Table 7. The complete description of these differences along with other additional criteria could be found in references 13, 14, 15, and 17.

Having calculated sleeper bending moments, bending strength of the sleeper should then be examined and controlled. AREMA [18] has developed a practical method for the estimation of ultimate bending capacities of sleepers. In this method, based on sleeper length, type of bending moment (i.e. positive or negative), and location of sleeper bending moment calculation (i.e. at rail seat or at sleeper mid-span), limit values are determined.

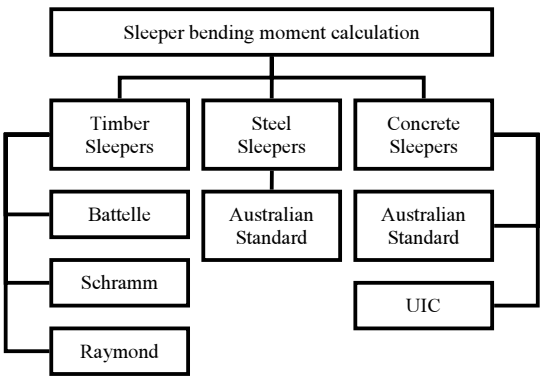



**Fig. 7.** Recommended methods for sleeper bending moment calculation

### 2.3. Rail Fastener

Rail fasteners also known as fastening systems are used in railway track structure to fasten the rails on the sleepers and to protect the rail from inadmissible vertical, lateral, and longitudinal movements as well as rail overturning. Moreover, these components serve as tools for gauge restraining, wheel load impact attenuation, increasing track elasticity, etc. There are enormous different types of rail fasteners which mainly are classified based on two important aspects as presented in Figure 8.

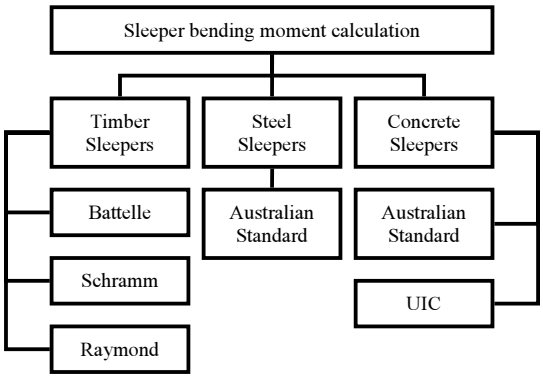



**Fig. 8.** Classification of rail fasteners

Despite the important roles rail fasteners play in track system, current practices in analysis and design of these track components are not extensively explained in the literature. Design criteria mentioned in many railway related design codes are restricted to those dealt with the acceptance criteria of laboratory qualification tests. AREMA manual [19], Australian Standard [20], and European Standards [21-25] are the most important design codes which include such criteria. A comparison between these standards is presented in Table 8.

### 2.4. Ballast and Sub-ballast Layers

Ballast and sub-ballast layers are composed of granular materials and used in track structure mainly to sustain the loads transferred from the sleeper. Other important functions of these layers include: 1) to reduce the stress intensity to the level to be tolerable for subgrade layer, 2) to absorb impact, noise and vibration induced from the wheels, 3) to restrict the track excessive settlement, 4) to facilitate track maintenance operations, especially those related to the correction of track geometry defects, and 5) to provide adequate drainage for the track structure, thereby track settlement as well as vegetation

**Table 7.** Comparison of different recommended methods for calculation of sleepers bending moments

| Sleeper Type | Developer | Rail Seat Moment | | Center Moment | |
|---|---|---|---|---|---|
| | | $M_r^+$ (kN.m) | $M_r^-$ (kN.m) | $M_c^+$ (kN.m) | $M_c^-$ (kN.m) |
| Timber | Battelle [17] | $q_r\left(\frac{l-g}{2}\right)$ (1) | *** | *** | $q_r\left(\frac{g}{2}\right)$ |
| | Schramm [17] | $q_r\left(\frac{l-g-n}{8}\right)$ | *** | *** | *** |
| | Raymond [17] | *** | *** | *** | $q_r\left(\frac{2g-l}{4}\right)$ |
| Steel | Australian Standard [15] | $q_r\left(\frac{l-g}{8}\right)$ | *** | $0.05\times q_r\times(l-g)$ | $q_r\left(\frac{2g-l}{4}\right)$ |
| Concrete | UIC [13] | $\gamma_i.q_r.\frac{\lambda}{2}$ (2) | $0.5M_r^+$ | $1.2M_{dr}+\times\frac{I_c}{I_r}$ | $0.7M_c^+$ |
| | Australian Standard [14] | $q_r\left(\frac{l-g}{8}\right)$ | Max{$0.67M_r^+$,14} | $0.05\times q_r\times(l-g)$ | $q_r\left(\frac{2g-l}{4}\right)$ |

(1) Less conservative and more realistic formula is also suggested by Battelle as $q_r\left(\frac{l-g}{8}\right)$

(2) The effective lever arm can be obtained from $\lambda=\frac{L_p-e}{2}$

growth will be limited. In addition to the above mentioned functions, the sub-ballast layer is used to act as a filter layer which prevents ballast and subgrade materials to be mixed together.

Current practices in structural analysis and design of ballast and sub-ballast are dealt with the determination of minimum required depth for theses granular layers. Theoretical, semi-empirical, and empirical methods have been developed and are being used in order to satisfy this design criterion.

Theoretical determination of minimum required depth based on the results of Boussinesq elastic theory applied to a uniform rectangular loaded area (see Figure 9) is performed using the numerical solution of the following equation:

$$\sigma_z=\frac{3P_a}{2\pi}\int_{\varepsilon=-A}^{\varepsilon=+A}\int_{\eta=-B}^{\eta=+B}\frac{d\varepsilon.d\eta}{\left\{(x-\varepsilon)^2+(y-\eta)^2+z^2\right\}^{5/2}} \quad (16)$$

It should be noted that, ballast and sub-ballast layers are assumed as a single homogenous and isotropic layer in Boussinesq elastic theory. Although such an assumption seems not to be sufficiently accurate, however, based on the comparison to the available field tests results, the ORE [26] investigations have proved the validity of Boussinesq elastic theory. This comparison is presented in Figure 10. It is apparent from this

**Table 8.** Comparison of different railway design codes with the consideration of rail fastener qualification tests

| Qualification Test | Design Code | | |
|---|---|---|---|
| | AREMA | AS 1085.19 | EN 13146 |
| Uplift restraint | ✓ | ✓ | ✓ |
| Longitudinal restraint | ✓ | ✓ | ✓ |
| Repeated load | ✓ | ✓ | × |
| Torsional restraint | ✓ | × | ✓ |
| Lateral restraint | × | ✓ | × |
| Clip spring rate | × | ✓ | × |
| Fatigue strength | × | ✓ | × |
| Impact attenuation | × | ✓ | ✓ |

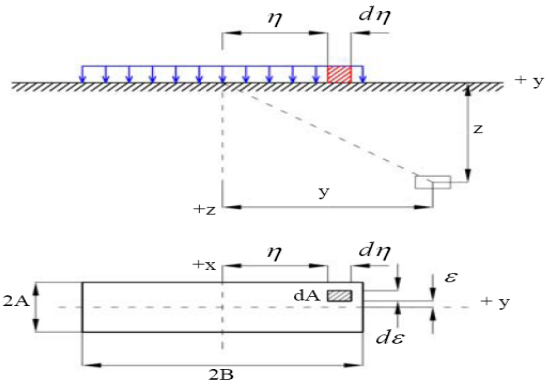



**Fig. 9.** Application of Boussinesq's elementary single vertical concentrated load over a uniformly loaded rectangular bearing area [17]

figure that the results obtained from Boussinesq method, reasonably remain within the envelope acquired from experimental investigations.

Simplified semi-empirical methods are also employed which assume that the load is distributed vertically with a load spread slope of 1 vertical to 1 horizontal or a slope of 2 vertical to 2 horizontal. Moreover, in this method, it is assumed that the stress distribution to be uniform at any given depth below the surface. This method calculates only the average vertical pressure at depth while the Boussinesq method calculates the maximum vertical pressure at a depth below the loaded area [17]. A comparison of the vertical stress distribution calculated for both 1:1 and 2:1 load spread assumptions with the theoretical Boussinesq solution is presented in Figure 11. It is clearly apparent that the assumed 2:1 load spread distribution of vertical pressure more closely approximates the Boussinesq pressure distribution than that of 1:1 load spread distribution.

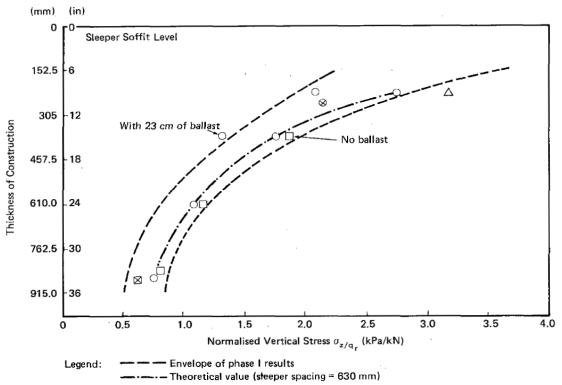



**Fig 10.** Comparison of experimental vertical stress distribution with depth and the Boussinesq solution [26]

Considering the 2:1 load spread distribution as indicated in Figure 12, the minimum required ballast and sub-ballast depth can be calculated from the following formula:

$$\sigma_z = P_a \frac{B.L}{(B+z)(L+z)} \qquad (17)$$

Having determined the allowable subgrade load carrying capacity and substituting it in the equations (16) or (17) the minimum required ballast and sub-ballast depth can be calculated.

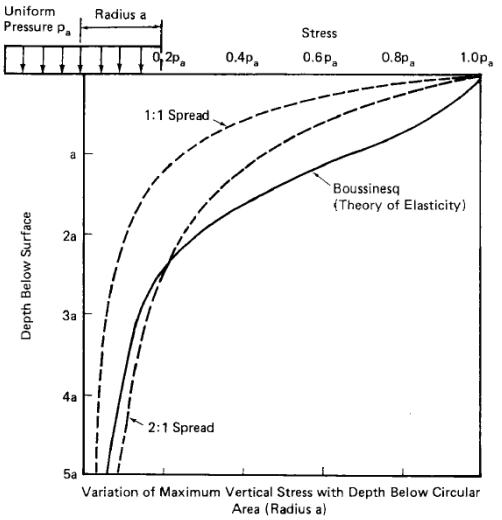



**Fig. 11.** Comparison of vertical stress distribution under a uniformly loaded circular area based on Boussinesq equations and 1:1 and 2:1 distributions [17]

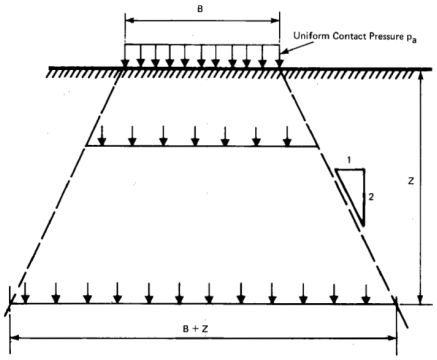



**Fig. 12.** Recommended semi-empirical pressure distribution in track supporting granular layers [17]

## 3. Discussion and Conclusions

Current practices of anlysis and design of railway track systems are entirely based on load bearing approach in which every track component is evaluated separately for its sufficient strength to sustain the environmental and traffic loads. Various analysis methods and design criteria have been reviewed in this paper. Based on comparisons of the available railway design methods, some shortcomings are discussed in this section.

Results of the main proposed dynamic impact factors presented in Table 1 are drawn against train speed in Figure 13. It is apparent from this figure that train speed can considerably influence

the values of dynamic impact factor obtained from different formulas. This result could be used to make suggestions on proper utilization of different methods used for the dynamic impact factor estimation. Since AREMA recommendations are basically in accordance to heavy haul operations in which trains usually pass over the track with lower speeds, it would not be practical to use the AREMA formula for the railway transit operations. Therefore, the use of the AREMA recommendation for dynamic impact factor is suggested for heavy haul railway with the train speed less that 80 km/h. On the other hand, the use of equation proposed by Eisenmann is more justifiable for the calculation of dynamic impact factor in high speed railway tracks. Although the consideration of dynamic impact factor somehow compensate for the effects of dynamic impact of the loads, the current railway design approach still needs further improvement by the consideration of transient characteristics of the wheel loads. Furthermore, the above review of the current approach indicates that the effect of higher train speeds is not included in the estimation of rail wear life. This urges the modification of current rail design criteria in accordance to high speed train operations.

The main proposed contact pressure between sleepers and ballast are presented in Table 4. As indicated in the previous section, a uniform contact pressure distribution is assumed between ballast and sleeper. This means that the current design criteria do not clearly include the effect of ballast material degradation on the ballast-sleeper pressure distribution pattern. In other words, the sleeper design approach awaits further improvements by the incorporation of long-term effects of the loads and in turn, consideration of the ballast degradation.

Current practices in analysis and design of railway tracks do not lead to a clear approach for the analysis and design of rail fasteners. The results of this study indicate that except the limited controlling criteria suggested by Australian Standard, there is no other obvious criteria proposed regarding rail fasteners' fatigue strength. Therefore, the procedure of the analysis and design of rail fasteners needs to be further

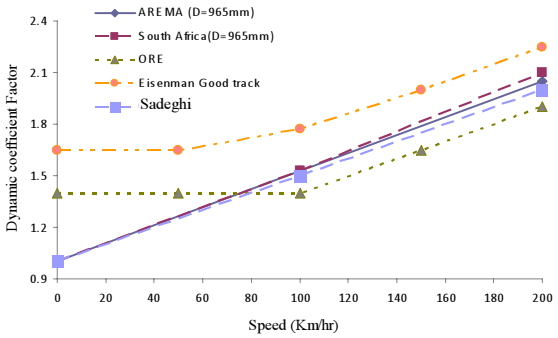



**Fig. 13.** Comparison of some recommended methods for the estimation of dynamic coefficient factor

developed. This importance would be more realized when considering the great variety of different rail fasteners currently used in the construction of railway track systems.

The only criterion determined for the structural analysis and design of granular ballast and sub-ballast supporting layers is the estimation of minimum required depth of these track components. This criterion is based on the amount of reduction in vertical pressure intensity to be tolerable for the underlying subgrade layer. However, there are no specific suggestions regarding the effect of the material gradation on optimum ballast and sub-ballast layers thickness determination. The influence of track maintenance operations as well as new construction techniques and facilities such as geosynthetics application, is not clearly included. The effect of accumulative loading and the consequent track plastic strain behavior is not incorporated clearly in track components analysis and design especially with respect to the sleepers as well as ballast and sub-ballast layers.

The results obtained in this research clearly indicate the need for further developments of current railway track design methods by more investigations of railway track short and long term behaviors with particular attention to the results obtained from laboratory and field tests. The aim is to define new criteria which include track parameters and loading conditions omitted in the current design approaches.

## List of Symbols

B Sleeper breadth, m

C height of the rail neutral axis from rail base,

mm

D Wheel diameter, mm

$D_A$ Annual gross tonnage, MGT/year

$D_c$ Degree of curve, degrees

$D_i$ Sub-tonnage, MGT/year

$D_j$ Track stiffness at the joint, kN/mm

D.F. Load distribution factor

e Width of the rail seat load distribution along sleeper thickness

E Rail modulus of elasticity, N/mm$^2$

$F_1$ Track support variation safety factor

$F_2$ Factor accounting for adjacent wheels interactions

$F_3$ Factor depending on the sleeper type and the standard of track maintenance

G Gravitational constant, m/s$^2$

I Rail moment of inertia, mm$^4$

$I_c$ Horizontal moment of inertia of the center of the sleeper cross section, mm$^4$

$I_r$ Horizontal moment of inertia of the sleeper cross section at rail seat position, mm$^4$

j Load amplification factor

$K_{A_i}$ Wheel load class factor

$K_C$ Track curvature and lubrication factor

$K_G$ Track gradient factor

$K_R$ Rail factor

$K_{S_i}$ Service type factor

$K_{V_i}$ Speed class factor

$K_w$ Wear factor varying with the degree of curve

l Sleeper length, m

$L_p$ Distance between rail seat axles and the end of the sleeper

n Length of steel rail plate

P Wheel load, kN

$P_s$ Static wheel load, kN

$P_u$ Unsprung weight of the wheel, kN

$R_w$ Wheel radius, mm

$R_{wt}$ Rail weight per unit length, kg/m

S Sleeper spacing, mm

T Rail wear life, MGT

$T_y$ Rail wear life, year

u Track modulus, N/mm$^2$

V Train speed, km/h

$W_t$ Average rail head area wear term, mm$^2$/MGT

**Greek symbols**

$\alpha$ Coefficient of expansion

$\alpha'$ Speed coefficient

$\beta'$ Speed coefficient

$\delta$ Factor related to track condition

$\Delta t$ Temperature variation

$\gamma'$ Speed coefficient

$\gamma_d$ Load distribution factor

$\gamma_i$ Dynamic increment of bending moment factor due to sleeper support irregularities

$\gamma_p$ Impact attenuation factor of rail fasteners

$\gamma_r$ Sleeper support condition factor

$\gamma_v$ Speed related amplification factor

$\eta$ Speed factor

$\lambda$ Effective lever arm, m

ν Poisson's ratio

**References**


[1] Kerr, A.D., "Fundamentals of Railway Track Engineering", Simmons-Boardman Books, Inc, 2003.

[2] Esveld, C., "Modern Railway Track", MRT-Productions, the Netherlands, 2001.

[3] Boresi, A.P., Schmidt, R.J., 2003, "Advanced Mechanics of Materials", 6th Edition. John Wiley & Sons, New York, NY: Chap. 5, 10.

[4] Sadeghi, J., and Yoldashkhan, M., 2005, "Investigation on the accuracy of current practices in analysis of railway track sleepers", International Journal of Civil Engineering, Vol. 3, No. 1, pp. 9-15.

[5] American Railway Engineering and Maintenance of way Association, Manual for Railway Engineering, Volume 4, Chapter 16, Part 10, "Economics of Railway Engineering and Operations – Construction and Maintenance Operations", 2006.

[6] Robnett, Q.L., Thompson, M.R., Hay, W.W., Tayabji, S.D., Peterson, H.C., Knutson, R.M., and Baugher, R.W., "Technical Data Bases Report", Ballast and Foundation Materials Research Program, FRA Report No. FRA-OR&D-76-138, National Technical Information Service, Springfield, Virginia, USA, 1975.

[7] Eisenmann, J., "Stress Distribution in the Permanent way due to Heavy Axle Loads and High Speeds", AREMA Proceedings, Vol. 71, pp. 24-59, 1970.

[8] Smith, J.O., and Liu, C.K., 1953, "Stresses Due to Tangential and Normal Loads on an Elastic Solid with Application to some Contact Stress Problems", Transactions of ASME, Journal of Applied Mechanics, pp. 157-166.

[9] Lundgren, J.R., Martin, G.C., and Hay, W.W., "A Simulation Model of Ballast Support and the Modulus of Track Elasticity", (Masters Thesis), Civil Engineering Studies, Transportation Series, No. 4, University of Illinios, 1970.

[10] Hay, W.W., Schuch, P.M., Frank, M.W., and Milskelsen, M.J., "Evaluation of Rail Sections", 2nd Progress Report, Civil Eng., Transportation Series No. 9, Univ. Illinois Project No. 44-22-20-332, Univ. Illinois, Urbana, 1973.

[11] Danzing J. C., Hay W., and Reinschmidt A., "Procedures for analyzing the economic costs of railway roadway for pricing purposes", Volume 1 and 2, Report No. RPD-11-CM-R, Tops on-line Service Inc, 1976.

[12] American Railway Engineering and Maintenance of Way Association, Manual for Railway Engineering, Volume 1, Chapter 30, Part 1, "Ties – General Considerations", 2006.

[13] International Union of Railways, "Design of Monoblock Concrete Sleepers", UIC CODE, 713 R, 1st Edition, 2004.

[14] Australian Standard, AS 1085.14, Railway track materials, Part 14: "Prestressed Concrete Sleepers", 2002.

[15] Australian Standard, AS 1085.17, Railway Track Materials, Part 17: "Steel Sleepers", 2002.

[16] Sadeghi, J., 2008, "Experimental Evaluation of Accuracy of Current Practices in Analysis and Design of Railway Track Sleepers", Canadian Journal of Civil Engineering. Vol. 35, pp. 881-893.

[17] Doyle, N.F., "Railway Track Design: A Review of Current Practice", Occasional Paper No. 35, Bureau of Transport Economics, Commonwealth of Australia, Canberra, 1980.

[18] American Railway Engineering and Maintenance of Way Association, Manual for Railway Engineering, Volume 1, Chapter 30, Part 4, "Ties – Concrete Ties". 2006.

[19] American Railway Engineering and Maintenance of Way Association, Manual for Railway Engineering, Volume 1, Chapter 5, Part 9, "Track – Design Qualification Specifications for Elastic Fasteners on Timber Cross Ties", 2006.

[20] Australian Standard, AS 1085.19, Railway track materials, Part 19: "Resilient fastening assemblies", 2002.

[21] European Committee for Standardization, EN 13146-1, Railway Applications – Track, Test Methods for Fastening Systems, Part 1: "Determination of Longitudinal Rail Restraint", 2002.

[22] European Committee for Standardization, EN 13146-2, Railway Applications – Track, Test Methods for Fastening Systems, Part 2: "Determination of Torsional Rail Resistance", 2002.

[23] European Committee for Standardization, EN 13146-3, Railway Applications – Track, Test Methods for Fastening Systems, Part 3: "Determination of Attenuation of Impact Loads", 2002.

[24] European Committee for Standardization, EN 13146-4, Railway Applications – Track, Test Methods for Fastening Systems, Part 4: "Effect of Repeated Loading", 2002.

[25] European Committee for Standardization, EN 13146-7, Railway Applications – Track, Test Methods for Fastening Systems, Part 7: "Determination of Clamping Force", 2002.

[26] Office of Research and Experiments (ORE), Stresses in the Formation, Question D17, "Stresses in the Rails, the Ballast and the