

Shape-size optimization of single-layer barrel vaults using improved magnetic charged system search

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Abstract

In this paper, the problem of simultaneous shape and size optimization of single-layer barrel vault frames which contains both of discrete and continuous variables is addressed. In this method, the improved magnetic charged system search (IMCSS) is utilized as the optimization algorithm and the open application programming interface (OAPI) plays the role of interfacing analysis software with the programming language. A comparison between the results of the present method and some existing algorithms confirms the high ability of this approach in simultaneous shape and size optimization of the practical and large-scale spatial structures.

Keywords: Shape-size optimization, Barrel vaults, Improved magnetic Charged system search, Open application programming interface.

1. Introduction

Optimization has become the most interesting and effective subject in the field of structural design. The presentation of many meta-heuristic algorithms in the last three decades is a continuing developmental process in the field of structural optimization. On the other hand, the increasing use of braced barrel vaults as a kind of light-weight space structures is inevitable. Optimization of barrel vaults, therefore, can be a highly effective issue in engineering design of these spatial structures.

Meta-heuristics algorithms are recent generation of the optimization approaches to solve complex problems. These methods explore the feasible region based on both randomization and some specified rules through a group of search agents. Laws of natural phenomena are usually source of the rules. Genetic Algorithm (GA) is introduced by Holland [1] and Goldberg [2]. It is inspired by biological evolutions theory. Particle swarm optimization (PSO) is introduced by Eberhart and Kennedy [3]. It simulates social behavior, and it is inspired by the migration of animals in a bird flock or fish school. The particle swarm algorithm is applied to truss optimization with dynamic constraints [4]. Ant Colony Optimization (ACO) is presented by Dorigo et al. [5].

It imitates foraging behavior of ant colonies. There are several other natural-inspired algorithms such as Simulated Annealing (SA) presented by Kirkpatrick et al. [6], Harmony Search (SA) introduced by Geem et al. [7], Big Bang–Big Crunch algorithm (BB–BC) presented by Erol and Eksin [8], and improved by Kaveh and Talatahari [9]. Due to good performance of these algorithms and their simple implementation, they have been widely used for solving various problems in different fields of science and engineering. One of the most recent meta-heuristic algorithms is the Charged System Search (CSS) proposed by Kaveh and Talatahari [10]. Electric laws of physics and the Newtonian laws of mechanics are used for guiding the Charged Particles (CPs) to explore the locations of the optimum. Colliding Bodies Optimization (CBO) is developed by Kaveh and Mahdavi [11] which is the first parameter independent method. Some applications of metaheuristic algorithms can be found in [12–14].

In the field of size optimization of single-layer barrel vaults frames some studies are carried out. Kaveh and Eftekhari have presented optimal design of barrel vault frames using IBB-BC algorithm [15], in which a 173-bar single layer barrel vault is optimized under both symmetrical and unsymmetrical load cases. In a study by the authors of present work, size optimization of some single layer barrel vault frames via IMCSS algorithm [16] has been presented.

In a study carried out by Parke [17], several different configurations of braced barrel vaults have been investigated using the stiffness method of analysis. Three different configurations have been analysed, each with five different span/height ratios and under both cases of symmetrical and non-symmetrical imposed nodal loads.

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The reported study which was a comparative investigation demonstrates that the most economical height-to-span ratio corresponding to the weight of the structures is approximately 0.17.

Some studies in the case of size optimization and a comparative study as the case of shape optimization are carried out for barrel vaults, but a more comprehensive study of the problem of simultaneous shape-size optimization of these structures is still needed. In this study, this problem is investigated using a new optimization approach. In this approach, a programming interface tool called OAPI is utilized and an improved version of a recently-proposed algorithm called IMCSS algorithm is used as the optimization problem.

A new meta-heuristic algorithm which is called Charged System Search (CSS) has been proposed by Kaveh and Talatahari [10]. This algorithm is based on the Coulomb and Gauss laws from physics and the governing laws of motion from the Newtonian mechanics. The modified version of CSS algorithm has been proposed by Kaveh et al. [19]. In MCSS algorithm, the magnetic laws are also considered in addition to electrical laws. In present paper IMCSS algorithm is utilized. In IMCSS algorithm to improve the performance of Harmony Search scheme in the algorithm and achieve more optimal results, some of the most effective parameters in convergence rate of algorithm are modified.

This paper has been organized as follows: in Section 2, problem of simultaneous shape and size optimization for barrel vault frames is expressed. Section 3 presents the optimization approach. In Section 4, the static loading conditions acting on the structures are defined. Two illustrative numerical examples are presented in Section 5 to examine the efficiency of the proposed approach, and finally in section 6, the concluding remarks are derived.

2. Statement of Optimization Problem for Barrel Vault Frames

The purpose of shape optimization of skeletal structures is to find a best state of nodal coordinates in order to minimize the weight of structure W , and on the other hand all of nodal coordinates of barrel vault structure are dependent to the height-to-span ratio. All of nodal coordinates, therefore, can be automatically calculated according to height in a constant span of barrel vault. In this process, the x and y coordinates of the joints will remain constant and the z coordinate of the nodes is calculated as follows:

$$z_i = \sqrt{R^2 - x_i^2} - (\sqrt{R-h}) \quad (1)$$

where x_i is the x coordinate of the i th joint and h is the height of barrel vault and R is the radius of semicircle which is expressed as

$$R = \frac{S^2 + 4h^2}{8h} \quad (2)$$

where S is the span of barrel vault.

The relation between nodal coordinates and height to span ratio for this type of space structures is depicted in Fig. 1.

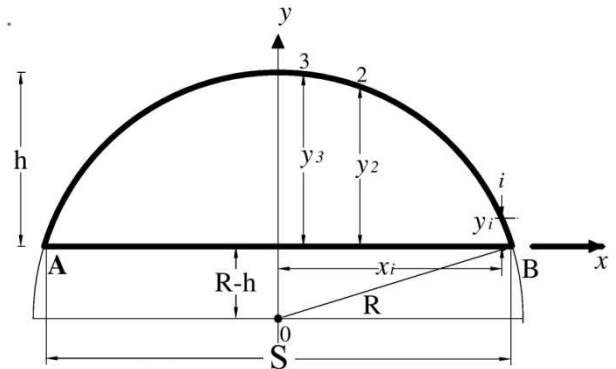


Fig. 1 The relation between nodal coordinates and height-to-span ratio (h/S) in the barrel vault

The aim of size of optimization of skeletal structures is to minimize the weight of structure W through finding the optimal cross-sectional areas A_i of members. All constraints exerted on the both problems of shape and size optimization must be satisfied, simultaneously.

According to the mentioned considerations, the problem of simultaneous shape and size optimization of barrel vault frames can be formulated as follows:

$$\begin{aligned} \text{Find} \quad & X = [x_1, x_2, x_3, \dots, x_n], h \\ & x_i \in \{d_1, d_2, \dots, d_{37}\}: \text{Discrete Variables} \\ & h_{\min} < h < h_{\max}: \text{Continuous Variable} \end{aligned} \quad (3)$$

$$\text{to minimize } \text{Mer}(X) = f_{\text{penalty}}(X) \times W(X)$$

Subjected to the following constraints
Displacement constraint:

$$v_i^d = \left| \frac{\delta_i}{\delta_i} \right| - 1 \leq 0, \quad i = 1, 2, \dots, nm \quad (4)$$

Shear constraint, for both major and minor axis (AISC-LRFD, Chapter G) [20]:

$$v_i^s = \frac{V_u}{\phi_v V_n} - 1 \leq 0, \quad i = 1, 2, \dots, nm \quad (5)$$

Constraints corresponding to interaction of flexure and axial force (AISC-LRFD, Chapter H) [20]:

$$v_i^I = \begin{cases} \frac{P_u}{2\phi_c P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) - 1 \leq 0 & \text{for } \frac{P_u}{\phi_c P_n} < 0.2 \\ \frac{P_u}{\phi_c P_n} + 8 \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) - 1 \leq 0 & \text{for } \frac{P_u}{\phi_c P_n} \geq 0.2 \end{cases}, \quad i = 1, 2, \dots, nm \quad (6)$$

where X is a vector which contains the design variables; for the discrete optimum design problem, the

variables x_i are selected from an allowable set of discrete values; n is the number of member groups; h is the height of barrel vault which is known as the only shape variable; d_j is the j th allowable discrete value for the size design variables; h_{min} , h_{max} are the permitted minimum and maximum values of the height which are respectively taken as $S/20$ and $S/2$ in this paper; S is the span of barrel vault; $Mer(X)$ is the merit function; $W(X)$ is the cost function, which is taken as the weight of the structure; $f_{penalty}(X)$ is the penalty function which results from the violations of the constraints corresponding to the response of the structure; nn is the number of nodes; δ_i , $\bar{\delta}_i$ are the displacement of the joints and the allowable displacement respectively; nm is the number of members; V_u is the required shear strength; V_n is the nominal shear strength which is defined by the equations in Chapter G of the LRFD Specification [20]; ϕ_v is the shear resistance factor $\phi_v = 0.9$; P_u is the required strength (tension or compression); P_n is the nominal axial strength (tension or compression); ϕ_c is the resistance factor ($\phi_c = 0.9$ for tension, $\phi_c = 0.85$ for compression); M_u is the required flexural strength; i.e., the moment due to the total factored load (Subscript x or y denotes the axis about which bending occurs.); M_n is the nominal flexural strength determined in accordance with the appropriate equations in Chapter F of the LRFD Specification [20] and ϕ_b is the flexural resistance reduction factor ($\phi_b = 0.9$).

For the displacement limitations which must be considered to ensure the serviceability requirements, the BS 5950 [21] limits the vertical deflections δ_v due to unfactored loads to $\text{Span}/360$, i.e. $\delta_v = S/360$ and horizontal displacements δ_H to $\text{Height}/300$, i.e. $\delta_H = h/300$ [22].

The nominal axial strength P_n is defined as:

$$P_n = A_g F_{cr} \quad (7)$$

where A_g is the gross area of member and F_{cr} is obtained as follows

$$F_{cr} = \begin{cases} (0.658^{\lambda_c^2}) F_y & \text{for } \lambda_c \leq 1.5 \\ \left(\frac{0.877}{\lambda_c^2} \right) F_y & \text{for } \lambda_c > 1.5 \end{cases} \quad (8)$$

where F_y is the specified minimum yield stress and the boundary between inelastic and elastic instability is $\lambda_c = 1.5$, where the parameter

$$\lambda_c = \frac{KL}{r\pi} \sqrt{\frac{F_y}{E}} \quad (9)$$

where K is the effective length factor for the member ($K = 1.0$ for braced frames [20]), L is the unbraced length of member, r is the governing radius of gyration about plane of buckling, and E is the modulus of elasticity for

the member of structure.

The cost function can be expressed as:

$$W(X) = \sum_{i=1}^{nm} \gamma_i \cdot x_i \cdot L_i \quad (10)$$

where γ_i is the material density of member i ; L_i is the length of member i ; and x_i is the cross-sectional area of member i as the design variable.

The penalty function can be defined as:

$$f_{penalty}(X) = \left(1 + \varepsilon_1 \cdot \sum_{j=1}^{np} v_{(j)}^k \right)^{\varepsilon_2} \quad (11)$$

where np is the number of multiple loading conditions. In this paper ε_1 is taken as unity and ε_2 is set to 1.5 in the first iterations of the search process, but gradually it is increased to 3 [23]. v^k is the summation of penalties for all imposed constraints for k th charged particle which is mathematically expressed as:

$$v^k = \sum_{i=1}^{nn} \max(v_i^d, 0) + \sum_{i=1}^{nm} (\max(v_i^I, 0) + \max(v_i^S, 0)) \quad (12)$$

where v_i^d , v_i^I , v_i^S are the summation of displacement, shear and interaction formula penalties which are calculated by Eqs. (4) through (6), respectively.

3. The Optimization Approach

An approach which contains improved magnetic charged system search (IMCSS) and open application programming interface (OAPI) is presented for the problem of simultaneous shape and size optimization of barrel vaults. The IMCSS is used as the optimization algorithm and the OAPI is utilized as an interface tool between analysis software and the programming language. In IMCSS algorithm, magnetic charged system search (MCSS) and an improved scheme of harmony search (IHS) are utilized, and two of the most effective parameters in the convergence rate of HS scheme are improved to achieve a good convergence rate and good solutions especially in final iterations [24].

The IMCSS algorithm and the OAPI tool are expressed in the following:

3.1. Improved magnetic charged system search

Recently, the CSS algorithm and its modified version, MCSS algorithm are respectively presented by Kaveh and Talathari [18] and Kaveh et al. [19] for optimization problems. The CSS algorithm takes its inspiration from the physic laws governing a group of Charged Particles (CPs). These charged particles are sources of the electric fields, and each CP can exert electric force on other CPs. The

movement of each CP due to the electric force can be determined using the Newtonian mechanic laws. The MCSS algorithm considers the magnetic force in addition to electric force for movement of CPs.

In this paper, an improved version of MCSS algorithm called IMCSS is presented. The IMCSS algorithm can be summarized as follows:

Level 1. Initialization

Step 1: Initialization. Initialize the algorithm parameters; the initial positions of CPs are determined randomly in the search space

$$x_{i,j}^{(0)} = x_{i,\min} + \text{rand} \cdot (x_{i,\max} - x_{i,\min}), \quad i = 1, 2, \dots, n. \quad (13)$$

where $x_{i,j}^{(0)}$ determines the initial value of the i th variable for the j th CP; $x_{i,\min}$ and $x_{i,\max}$ are the minimum and the maximum allowable values for the i th variable; rand is a random number in the interval [0,1]; and n is the number of variables. The initial velocities of charged particles are zero

$$v_{i,j}^{(0)} = 0, \quad i = 1, 2, \dots, n. \quad (14)$$

The magnitude of the charge is calculated as follows:

$$q_i = \frac{\text{fit}(i) - \text{fitworst}}{\text{fitbest} - \text{fitworst}}, \quad i = 1, 2, \dots, N. \quad (15)$$

where fitbest and fitworst are the best and the worst fitness of all particles; fit(i) represents the fitness of the agent i ; and N is the total number of CPs. The separation distance r_{ij} between two charged particles is defined as:

$$r_{ij} = \frac{\|X_i - X_j\|}{\|(X_i + X_j)/2 - X_{\text{best}}\| + \varepsilon}, \quad (16)$$

where X_i and X_j are the positions of the i th and j th CPs, X_{best} is the position of the best current CP, and ε is a small positive number to avoid singularities.

Step 2. CP ranking. Evaluate the values of Merit function for the CPs, compare with each other and sort them in an increasing order based on the corresponding value of merit function.

Step 3. Creation of Charged Memory (CM). Store CMS number of the first CPs in the CM.

Level 2: Search

Step 1: Force calculation. The probability of the attraction of the i th CP by the j th CP is expressed as:

$$p_{ij} = \begin{cases} 1 & \frac{\text{fit}(i) - \text{fitbest}}{\text{fit}(j) - \text{fit}(i)} > \text{rand} \text{ or } \text{fit}(j) > \text{fit}(i), \\ 0 & \text{else.} \end{cases} \quad (17)$$

where rand is a random number which is uniformly distributed in the range of (0,1). The resultant electrical

force $F_{E,j}$ acting on the j th CP can be calculated as follow:

$$F_{E,j} = q_j \cdot \sum_{i,i \neq j} \left(\frac{q_i}{a^3} r_{ij} \cdot i_1 + \frac{q_i}{r_{ij}^2} i_2 \right) \cdot p_{ij} (X_i - X_j), \quad \begin{cases} i_1 = 1, i_2 = 0 \Leftrightarrow r_{ij} < a, \\ i_1 = 0, i_2 = 1 \Leftrightarrow r_{ij} \geq a, \\ j = 1, 2, \dots, N. \end{cases} \quad (18)$$

The probability of the magnetic influence (attracting or repelling) of the i th wire (CP) on the j th CP is expressed as:

$$pm_{ij} = \begin{cases} 1 & \text{fit}(j) > \text{fit}(i), \\ 0 & \text{else.} \end{cases} \quad (19)$$

where fit(i) and fit(j) are the objective values of the i th and j th CP, respectively. This probability determines that only a good CP can affect a bad CP by the magnetic force.

The resultant magnetic force $F_{B,j}$ acting on the j th CP due to the magnetic field of the i th virtual wire (i th CP) can be expressed as:

$$F_{B,j} = q_j \cdot \sum_{i,i \neq j} \left(\frac{I_i}{R^2} r_{ij} \cdot z_1 + \frac{I_i}{r_{ij}} z_2 \right) \cdot pm_{ij} (X_i - X_j), \quad \begin{cases} z_1 = 1, z_2 = 0 \Leftrightarrow r_{ij} < R, \\ z_1 = 0, z_2 = 1 \Leftrightarrow r_{ij} \geq R, \\ j = 1, 2, \dots, N. \end{cases} \quad (20)$$

where q_i is the charge of the i th CP, R is the radius of the virtual wires, I_i is the average electric current in each wire, and pm_{ij} is the probability of the magnetic influence (attracting or repelling) of the i th wire (CP) on the j th CP.

The average electric current in each wire I_i can be expressed as:

$$(I_{\text{avg}})_{ik} = \text{sign}(df_{i,k}) \times \frac{|df_{i,k}| - df_{\min,k}}{df_{\max,k} - df_{\min,k}}, \quad (21)$$

$$df_{i,k} = \text{fit}_k(i) - \text{fit}_{k-1}(i), \quad (22)$$

where $df_{i,k}$ is the variation of the objective function of the i th CP in the k th movement (iteration). Here, $\text{fit}_k(i)$ and $\text{fit}_{k-1}(i)$ are the values of the objective function of the i th CP at the start of the k th and $k-1$ th iterations, respectively. Considering absolute values of $df_{i,k}$ for all of the current CPs, $df_{\max,k}$ and $df_{\min,k}$ would be the maximum and minimum values among these absolute values of df , respectively.

A modification can be considered to avoid trapping in part of search space (Local optima) because of attractive electrical force in CSS algorithm [19]

$$F = p_r \times F_E + F_B, \quad (23)$$

where p_r is the probability that an electrical force is a repelling force which is defined as

$$p_r = \begin{cases} 1 & \text{rand} > 0.1 \cdot (1 - \text{iter} / \text{iter}_{\max}), \\ -1 & \text{else.} \end{cases} \quad (24)$$

where rand is a random number uniformly distributed in the range of (0,1), iter is the current number of iterations, and $\text{iter}_{\max}$ is the maximum number of iterations.

Step 2: Obtaining new solutions. Move each CP to the new position and calculate the new velocity as follows:

$$X_{j,new} = rand_{j1} \cdot k_a \cdot \frac{F_j}{m_j} \cdot \Delta t^2 + rand_{j2} \cdot k_v \cdot V_{j,old} \cdot \Delta t + X_{j,old}, \quad (25)$$

$$V_{j,new} = \frac{X_{j,new} - X_{j,old}}{\Delta t}, \quad (26)$$

where $rand_{j1}$ and $rand_{j2}$ are two random numbers uniformly distributed in the range of (0,1). Here, m_j is the mass of the j th CP which is equal to q_j . Δt is the time step and is set to unity. k_a is the acceleration coefficient; k_v is the velocity coefficient to control the influence of the previous velocity. k_a and k_v are considered as:

$$k_a = c_1 \cdot \left(1 + \frac{iter}{iter_{max}}\right), \quad (27)$$

$$k_v = c_2 \cdot \left(1 - \frac{iter}{iter_{max}}\right),$$

where c_1 and c_2 are two constants to control the exploitation and exploration of the algorithm, respectively.

Step 3. Position correction of CPs. If each CP violates from its allowable boundary, its position is corrected using an improved harmony search-based approach which is expressed as follow:

In the process of position correction of CPs using harmony search-based approach, the CMCR and PAR parameters help the algorithm to find globally and locally improved solutions, respectively. PAR and bw in HS scheme are very important parameters in fine-tuning of optimized solution vectors, and can be potentially useful in adjusting convergence rate of algorithm to reach more optimal solution [25]. The standard version of CSS and MCSS algorithms, use the traditional HS scheme with constant values for both PAR and bw. Small PAR values with large bw values can led to poor performance of the algorithm and considerable increase in iterations needed to find optimum solution. Although small bw values in final iterations increase the fine-tuning of solution vectors, but

in the first iterations bw must take a bigger value to enforce the algorithm to increase the diversity of solution vectors. Furthermore, large PAR values with small bw values usually led to improvement of the best solutions in final iterations a better convergence to optimal solution vector. To improve the performance of the HS scheme and eliminate the drawbacks lies with constant values of PAR and bw, the IMCSS algorithms uses improved HS scheme with the variable values of PAR and bw in position correction step. PAR and bw change dynamically with iteration number as shown in. 2 and expressed as follow [25]:

$$PAR(iter) = PAR_{min} + \frac{(PAR_{max} - PAR_{min})}{iter_{max}} \cdot iter \quad (28)$$

and

$$bw(iter) = bw_{max} \exp(c \cdot iter), \quad (29)$$

$$c = \frac{\ln\left(\frac{bw_{min}}{bw_{max}}\right)}{iter_{max}}, \quad (30)$$

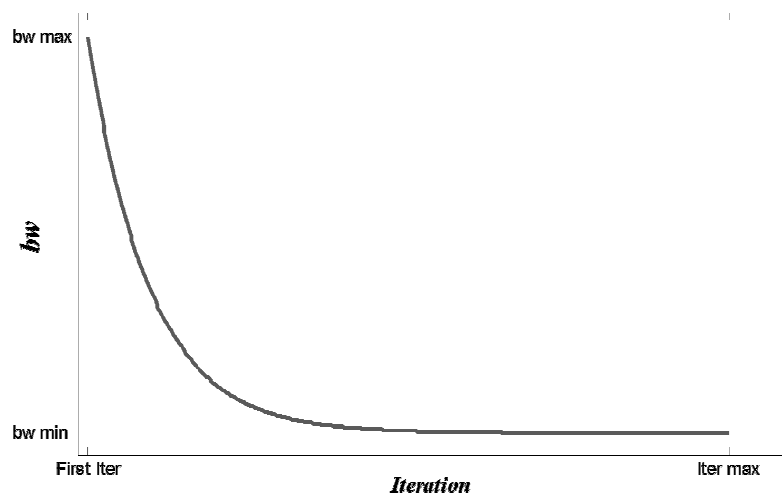
where $PAR(iter)$ and $bw(iter)$ are the values of PAR and bandwidth for current iteration, respectively. bw_{min} and bw_{max} are the minimum and maximum bandwidth, respectively.

Step 4: CP ranking. Evaluate and compare the values of Merit function for the new CPs, and sort them in an increasing order.

Step 5: CM updating. If some new CP vectors are better than the worst ones in the CM (means better corresponding Merit function), include the better vectors in the CM and exclude the worst ones from the CM.

Level 3: Controlling the terminating criterion.

Repeat the search level steps until a terminating criterion is satisfied. The terminating criterion is considered to be the number of iterations.



(a)

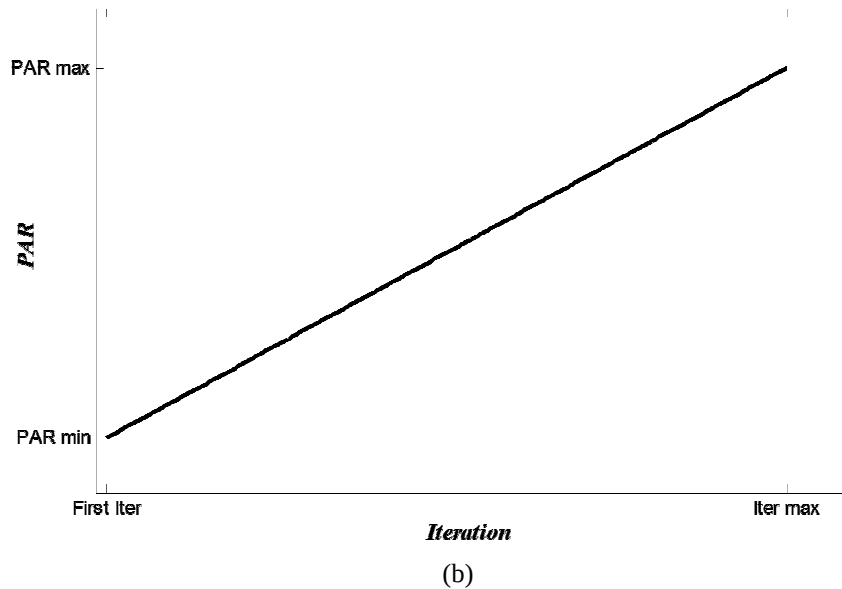


Fig. 2 Variation of (a) *bw* and (b) *PAR* versus iteration number in IMCSS algorithm [16]

3.1.1. Discrete IMCSS algorithm

The present algorithms can be also applied to optimal design problem with discrete variables. One way to solve discrete problems using a continuous algorithm is to utilize a rounding function which changes the magnitude of a result to the nearest discrete value, as follow

$$X_{j,new} = \text{Fix} \left(\text{rand}_{j1} \cdot k_a \cdot \frac{F_j}{m_j} \cdot \Delta t^2 + \text{rand}_{j2} \cdot k_v \cdot V_{j,old} \cdot \Delta t + X_{j,old} \right), \quad (31)$$

where $\text{Fix}(X)$ is a function which rounds each elements of vector X to the nearest permissible discrete value. Using this position updating formula, the agents will be permitted to select discrete values [26].

3.2. Open application programming interface

Recently, Computers and Structures Inc. introduces a powerful interface tool known as Open Application Programming Interface (OAPI). The OAPI can be utilized to automate and manage many of the processes required to build, analyze and design models through a programming language [27].

The computer program SAP2000 is a software of proven ability in analysis and design of practical large-scale structures. The utilization of this software, therefore, could be useful for the problem of structural optimization. In this process, the OAPI can be utilized in order to interface SAP2000 with the programming language which provides a path for two-way exchange of SAP model information with the programming language. There are many programming languages can be used to access SAP2000 through the OAPI such as MATLAB, Visual Basic, Visual C#, Intel Visual Fortran, Microsoft Visual C++ and Python.

In some studies carried out by the authors of this paper,

size optimization of single layer barrel vault frames [16] and double layer barrel vaults [13] is already investigated using this interface tool and MATLAB. Furthermore, Kaveh et al. [25] have utilized this interfacing ability in the form of parallel computing within the MATLAB for practical optimum design of real size 3D steel frames.

In this paper, the language of technical computing MATLAB is utilized in order for performing the process of optimization via presented approach (OAPI and IMCSS).

4. Static Loading Conditions

According to ANSI-A58.1 [29] and ASCE/SEI 7-10 [30] codes, there are some specific considerations for loading conditions of arched roofs such as barrel vault structures. In this paper, three static loading conditions are considered for optimization of these structures which are expressed as follows:

4.1. Dead load (DL)

A uniform dead load of 100 kg/m^2 is considered for estimated weight of sheeting, space frame, and nodes of barrel vault structure.

4.2. Snow load (SL)

The snow load for arched roofs is calculated according to ANSI [29] and ASCE [30] codes. Snow loads acting on a sloping surface shall be assumed to act on the horizontal projection of that surface. The sloped roof (balanced) snow load, P_s , shall be obtained by multiplying the flat roof snow load, P_f , by the roof slope factor, C_s , as follows:

$$P_s = C_s \cdot P_f \quad (32)$$

where C_s is

$$C_s = \begin{cases} 1.0 & \alpha < 15^\circ \\ 1.0 - \frac{\alpha - 15}{60} & 15^\circ < \alpha < 60^\circ \\ 0.25 & \alpha > 60^\circ \end{cases} \quad (33)$$

The C_s distribution in arched roofs is shown in Fig. 3. In this paper, the flat roof snow load P_f is set to 150 kg/m^2 .

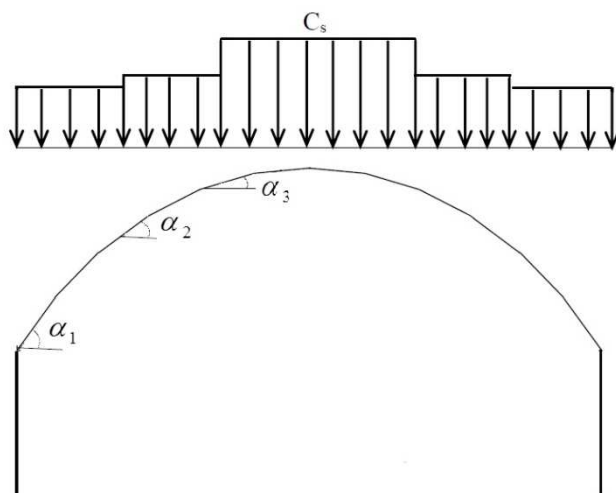


Fig. 3 C_s distribution in arched roofs

4.3. Wind load (WL)

For wind load in arched roofs, different loads are applied in the windward quarter, Center half and leeward quarter of the roof which are computed based on ANSI [29] and ASCE [30] codes as

$$P = q G_h C_p \quad (34)$$

where q is the wind velocity pressure, G_h is gust-effect factor and C_p is the external pressure coefficient. These parameters are calculated according to ANSI [29] and ASCE [30] codes.

5. Numerical Examples

This study presents optimal shape and size design of two single layer barrel vault frames which are first provided for size optimization by Kaveh et al. [16]. For all of examples a population of 100 charged particles is used and the value of $CMCR$ is set to 0.95. The values of PAR_{min} and PAR_{max} in IMCSS algorithm are set to 0.35 and 0.9, respectively.

The two examples are discrete optimum design problems and the variables are selected from an allowable set of steel pipe sections taken from AISC-LRFD code [31] shown in Table 1. For analysis of these structures, SAP2000 is used through OAPI tool and the optimization process is performed in MATLAB.

Table 1 The allowable steel pipe sections taken from AISC-LRFD code [31]

	Section Name	Dimensions	Weight per ft (lb)	Properties				
		Nominal Diameter (in.)		Area (in. ²)	Moment of inertia (in. ⁴)	Elastic section modulus (in. ³)	Gyration radius (in)	Plastic section modulus (in. ³)
1	P0.5	½	0.85	0.25	0.017	0.041	0.261	0.059
2	P0.75	¾	1.13	0.333	0.037	0.071	0.334	0.1
3	P1	1	1.68	0.494	0.087	0.133	0.421	0.187
4	P1.25	1 ¼	2.27	0.669	0.195	0.235	0.54	0.324
5	P1.5	1 ½	2.72	0.799	0.31	0.326	0.623	0.448
6	P10	2	3.65	1.07	0.666	0.561	0.787	0.761
7	P12	2 ½	5.79	1.7	1.53	1.06	0.947	1.45
8	P2	3	7.58	2.23	3.02	1.72	1.16	2.33
9	P2.5	3 ½	9.11	2.68	4.79	2.39	1.34	3.22
10	P3	4	10.79	3.17	7.23	3.21	1.51	4.31
11	P3.5	5	14.62	4.3	15.2	5.45	1.88	7.27
12	P4	6	18.97	5.58	28.1	8.5	2.25	11.2
13	P5	8	28.55	8.4	72.5	16.8	2.94	22.2
14	P6	10	40.48	11.9	161	29.9	3.67	39.4
15	P8	12	49.56	14.6	279	43.8	4.38	57.4
16	XP0.5	½	1.09	0.32	0.02	0.048	0.25	0.072
17	XP0.75	¾	1.47	0.433	0.045	0.085	0.321	0.125
18	XP1	1	2.17	0.639	0.106	0.161	0.407	0.233
19	XP1.25	1 ¼	3	0.881	0.242	0.291	0.524	0.414
20	XP1.5	1 ½	3.63	1.07	0.391	0.412	0.605	0.581
21	XP10	2	5.02	1.48	0.868	0.731	0.766	1.02
22	XP12	2 ½	7.66	2.25	1.92	1.34	0.924	1.87

23	XP2	3	10.25	3.02	3.89	2.23	1.14	3.08
24	XP2.5	3 ½	12.5	3.68	6.28	3.14	1.31	4.32
25	XP3	4	14.98	4.41	9.61	4.27	1.48	5.85
26	XP3.5	5	20.78	6.11	20.7	7.43	1.84	10.1
27	XP4	6	28.57	8.4	40.5	12.2	2.19	16.6
28	XP5	8	43.39	12.8	106	24.5	2.88	33
29	XP6	10	54.74	16.1	212	39.4	3.63	52.6
30	XP8	12	65.42	19.2	362	56.7	4.33	75.1
31	XXP2	2	9.03	2.66	1.31	1.1	0.703	1.67
32	XXP2.5	2 ½	13.69	4.03	2.87	2	0.844	3.04
33	XXP3	3	18.58	5.47	5.99	3.42	1.05	5.12
34	XXP4	4	27.54	8.1	15.3	6.79	1.37	9.97
35	XXP5	5	38.59	11.3	33.6	12.1	1.72	17.5
36	XXP6	6	53.16	15.6	66.3	20	2.06	28.9
37	XXP8	8	72.42	21.3	162	37.6	2.76	52.8

In all examples, the material density is 0.2836 lb/in³ (7850 kg/m³) and the modulus of elasticity is 30450 ksi (2.1×10⁶ kg/cm²). The yield stress F_y of steel is taken as 34135.96 psi (2400 kg/cm²) for both problems.

5.1. A 173-bar single-layer barrel vault frame

The 173-bar single layer barrel vault frame with a 2-way grid pattern is shown in Fig. 4. This spatial structure consists of 108 joints and 173 members. There are 16 design variables in this problem which consist of size and shape variables. For the process of size optimization, all members of this structure are categorized into 15 groups, as shown in Fig. 4(b). Furthermore, for the problem of shape optimization, the lower and upper bounds of height as the only shape variable are 1.5 m and 15 m, respectively. The nodal displacements are limited to ±1.05 in (26 mm) in x, y

directions and ±1.64 in (41 mm) in z direction.

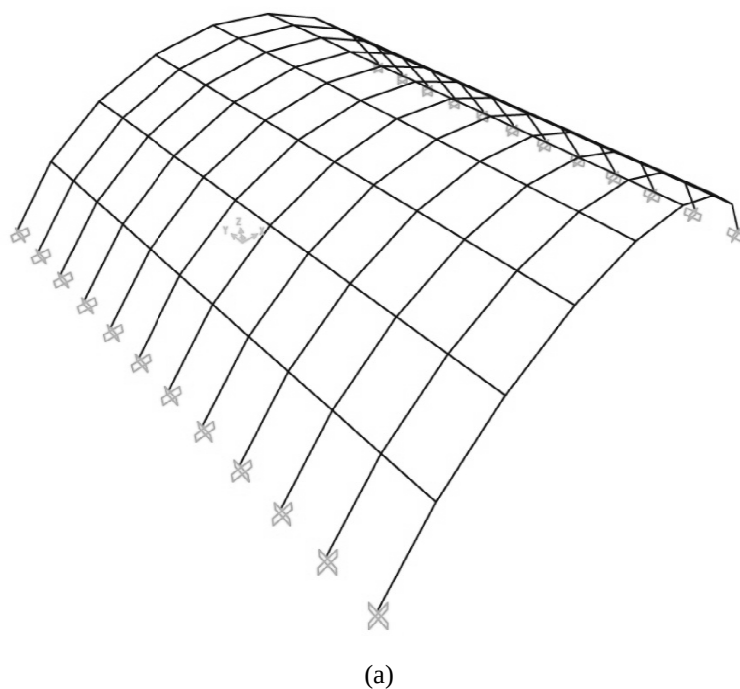
The configuration of the 173-bar single layer barrel vault is as follows

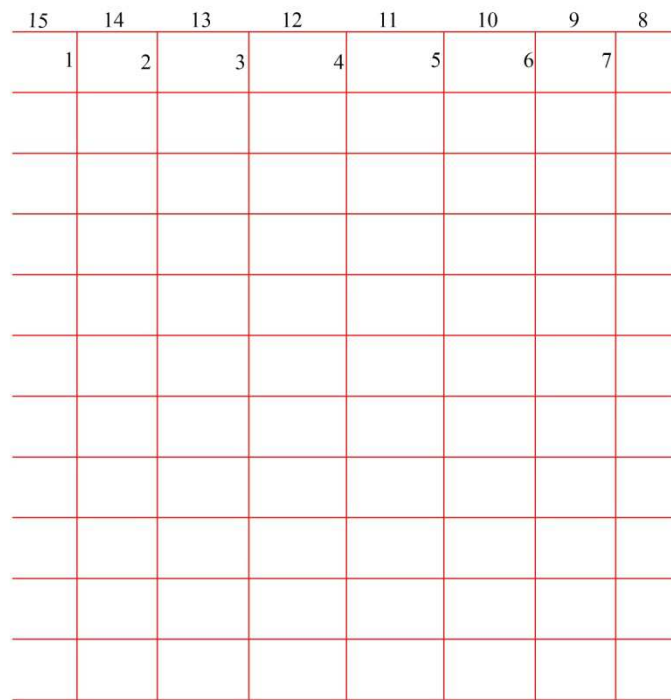
- Span (S) = 30 m (1181.1 in)
- Height (H) = 8 m (314.96 in)
- Length (L) = 30 m (1181.1 in)

According to ANSI/ASCE considerations mentioned in Section 4, this spatial structure is subjected to three loading conditions:

A uniform dead load of 100 kg/m² is applied on the roof. The applied snow and wind loads on this structure are shown in Fig. 5 (a) and (b), respectively.

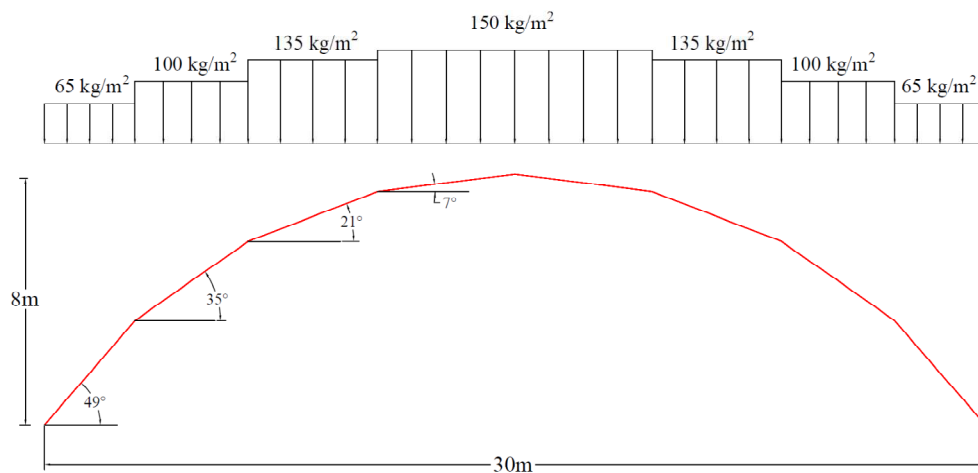
The convergence history for optimization of this structure using CSS, MCSS and IMCSS algorithms is shown in Fig. 5. Comparison of the optimal design results using presented algorithms is also provided in Table 2.



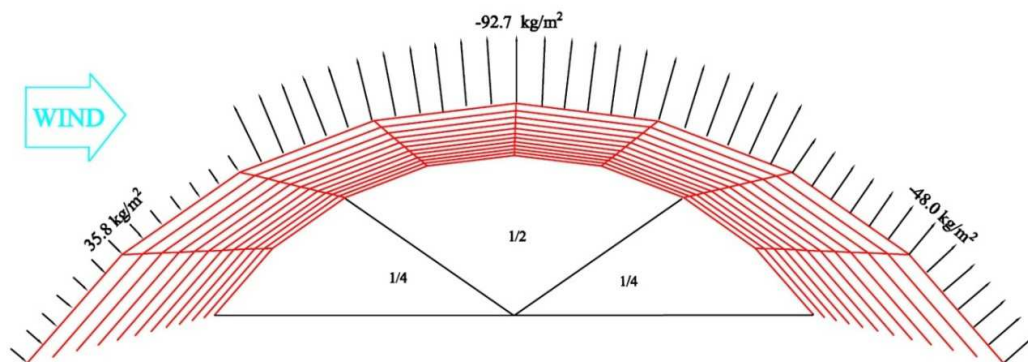


(b)

Fig. 4 The 173-bar single layer barrel vault frame, (a) 3-dimensional view, (b) Member groups in top view [16]



(a)

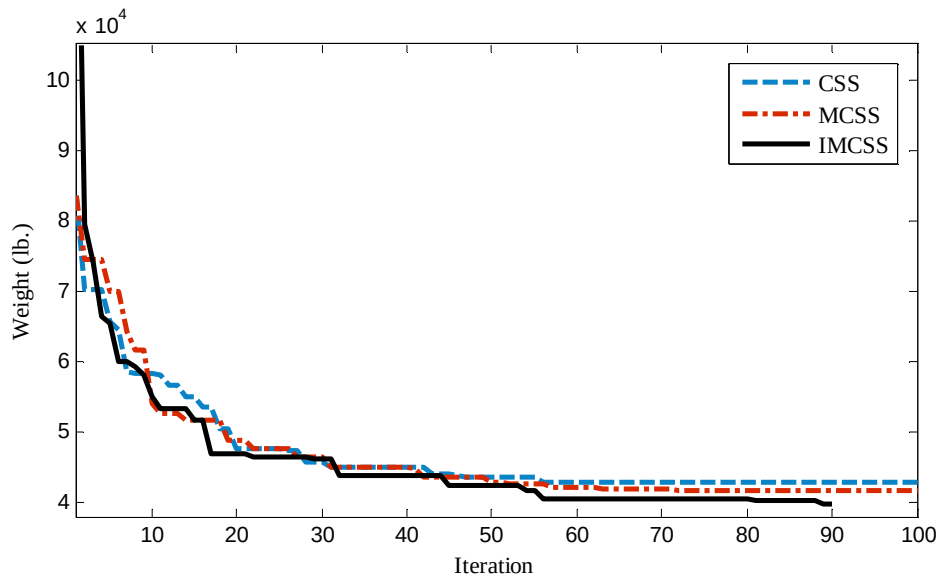


(b)

Fig. 5 The 173-bar single layer barrel vault frame subjected to: (a) Snow and (b) Wind loading [16]

Table 2 Optimal solutions for simultaneous shape and size optimization of the 173-bar barrel vault (in²)

Design Variables		CSS		MCSS		IMCSS	
		Section Name	Area (in. ²)	Section Name	Area (in. ²)	Section Name	Area (in. ²)
1	A1	'XP1'	0.639	'XP1'	0.639	P1	0.494
2	A2	'XP1.5'	1.07	'XP1.25'	0.881	P2.5	1.7
3	A3	'XXP2'	2.66	'P2.5'	1.7	XP1.5	1.07
4	A4	'P1.5'	0.799	'XP2'	1.48	P3	2.23
5	A5	'P3.5'	2.68	'XP1.5'	1.07	XP1.5	1.07
6	A6	'XP1.25'	0.881	'P2.5'	1.7	P1.5	0.799
7	A7	'XP2'	1.48	'P1.5'	0.799	P1	0.494
8	A8	'P10'	11.9	'P10'	11.9	P10	11.9
9	A9	'XP6'	8.4	'XP6'	8.4	XP6	8.4
10	A10	'XP6'	8.4	'P10'	11.9	XP6	8.4
11	A11	'P10'	11.9	'XP6'	8.4	P10	11.9
12	A12	'XP6'	8.4	'P10'	11.9	P10	11.9
13	A13	'XP6'	8.4	'P6'	5.58	P6	5.58
14	A14	'P6'	5.58	'P6'	5.58	P6	5.58
15	A15	'P12'	14.6	'P10'	11.9	XP6	8.4
16	Height	131.0308 in (3.33 m)		132.6162 in (3.37 m)		113.9046 in (2.89 m)	
Weight. lb.		42957.98		41589.25		39778.21	
Weight. Kg.		19485.41		18864.57		18043.09	
Max. Displacement (in)		1.6118		1.4360		1.1277	
Max. Strength Ratio		0.9865		0.9604		0.9516	
No. of Analyses		10,000		10,000		8,900	

**Fig. 6** Convergence history for the 173-bar single layer barrel vault frame using CSS, MCSS and IMCSS algorithms

As seen in Table 2, the IMCSS algorithm finds its best solutions in 89 iterations (8,900 analyses), but the CSS and MCSS algorithms have not found any better solutions in 10,000 analyses. The best weights of IMCSS is 39778.21 lb (18043.09 kg), while it is 41589.25 lb and 42957.98 lb for the MCSS and CSS algorithms, respectively. As it can be seen in the results, the IMCSS algorithm obtain a better weight in a lower number of analyses than previous algorithms.

Furthermore, the values of 131.03 in, 131.62 in and 113.9 in are obtained for the height of barrel vault from the CSS, MCSS and IMCSS algorithms, respectively. Hence, the best height-to-span ratios obtained from CSS, MCSS and IMCSS are 0.11, 0.11 and 0.10, respectively. It can be seen that, these values are approximately close to ratio of

0.17 from Parke's study. As seen in Table 2, the maximum strength ratio for CSS, MCSS and IMCSS algorithms is 0.9865, 0.9604 and 0.9516 respectively, and the maximum displacement is 1.6118 in, 1.4360 in and 1.1277 in for the CSS, MCSS and IMCSS algorithms, respectively.

Fig. 7 (a) to (c) provide strength ratios for all elements of the 173-bar single layer barrel vault frame for optimal results of CSS, MCSS and IMCSS algorithms, respectively. The figures show that all strength ratios of elements are lower than 1, thus there is no violation of constraints in the optimal results of presented algorithms and all strength constraints are satisfied. The maximum strength ratios for element groups of the 173-bar single layer barrel vault frame is shown in Fig. 8(a) through (c) for optimal results of the presented algorithms.

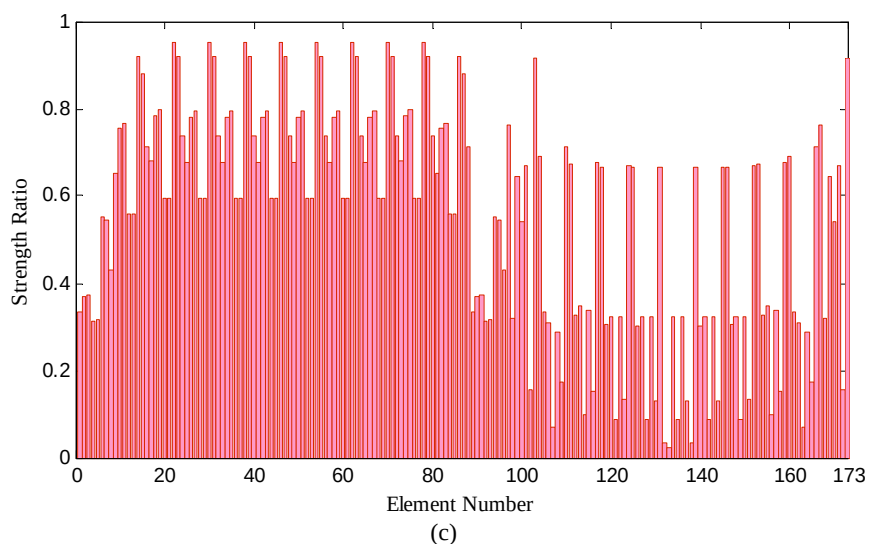
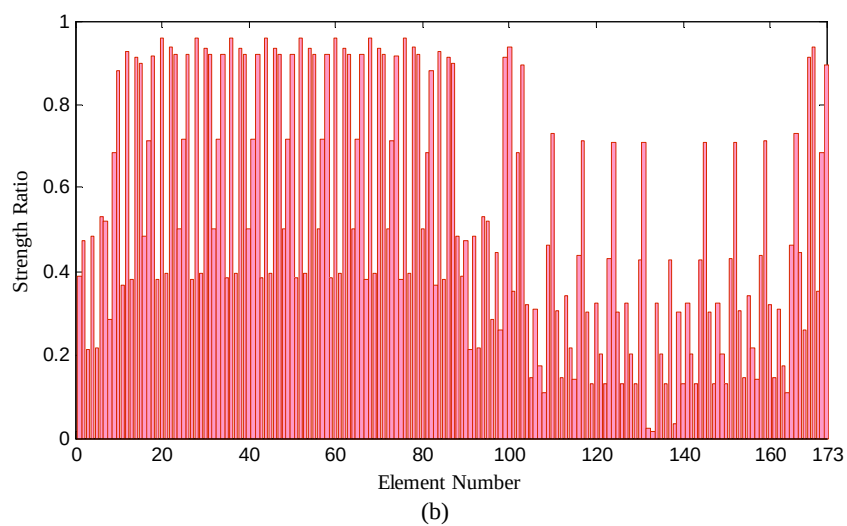
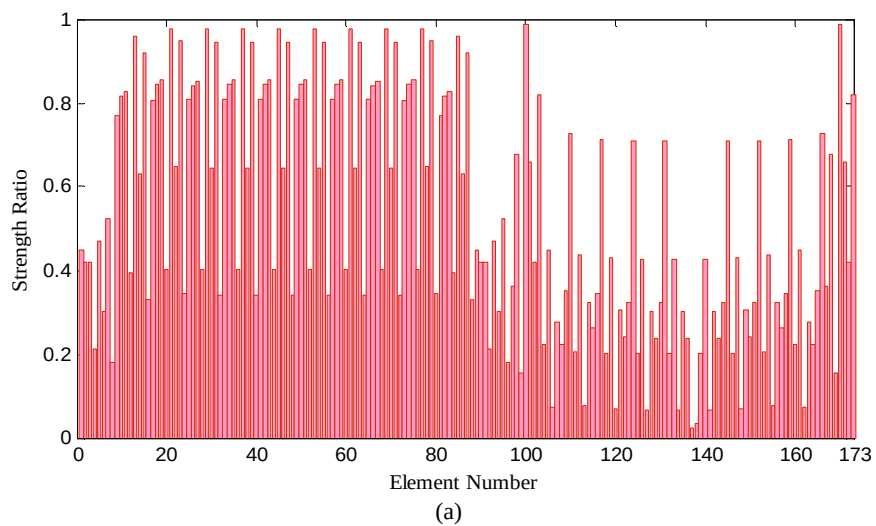


Fig. 7 Strength ratios for the elements of the 173-bar single layer barrel vault frame for optimal results of (a) CSS, (b) MCSS and (c) IMCSS algorithms

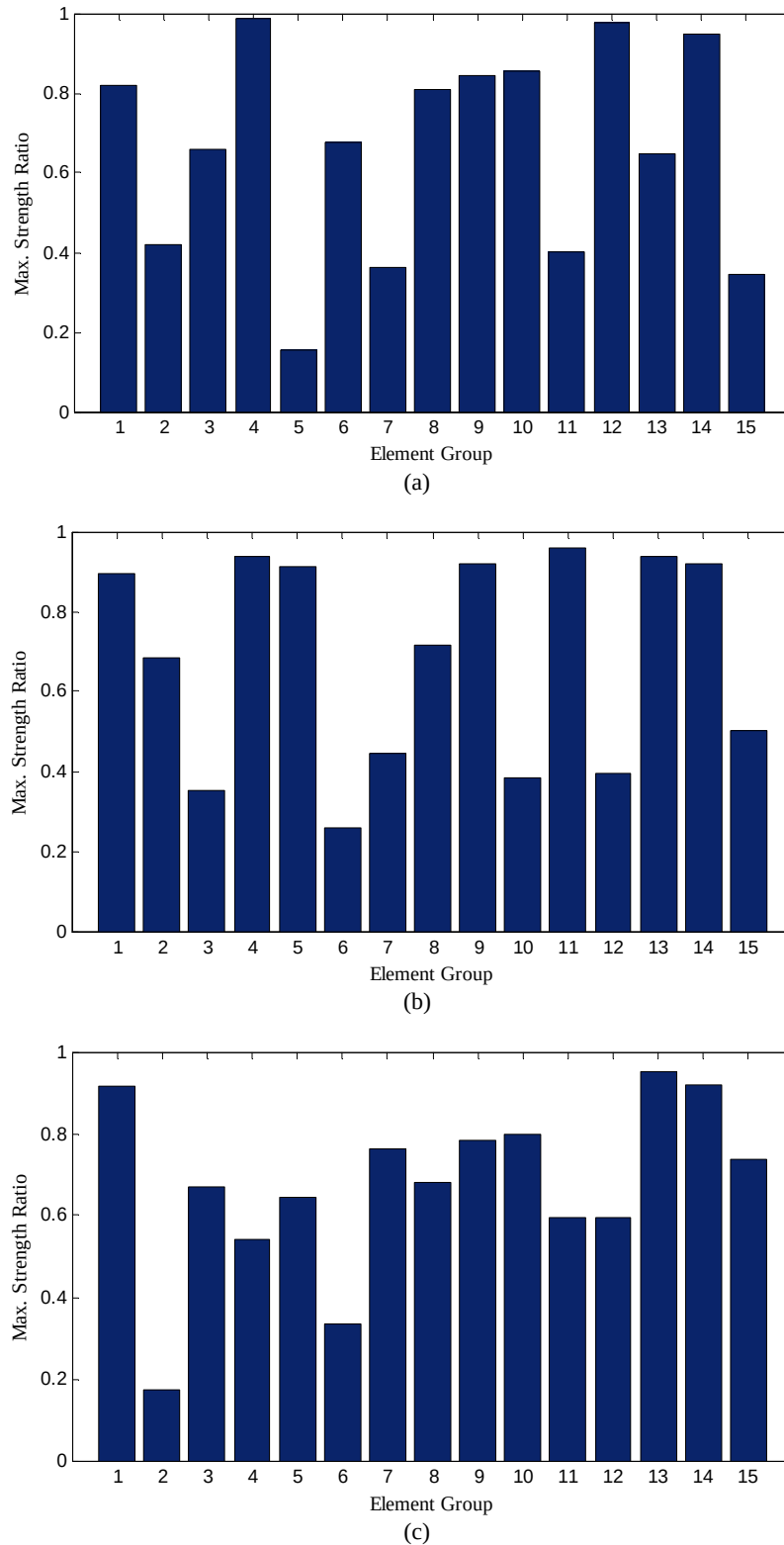


Fig. 8 Maximum strength ratios for element groups of the 173-bar single layer barrel vault frame for optimal results of (a) CSS, (b) MCSS and (c) IMCSS algorithms

Table 3 provides a comparison for the results of present work on simultaneous shape and size optimization with those of previous study [16] on size optimization of the 173-bar barrel vault. Comparison of best weight for both problem is also shown in Table 4. As it can be seen in

the results, the value of weight of structure have been reduced by 14.59%, 17.23% and 18.8% via CSS, MCSS and IMCSS algorithms, respectively.

Table 3 Comparison of the optimal solutions for the 173-bar single layer barrel vault frame

Design Variables		Kaveh <i>et al.</i> [16]			Present Work		
		Size Optimization			Simultaneous Shape and Size Optimization		
		CSS	MCSS	IMCSS	CSS	MCSS	IMCSS
1	A1	0.494	0.639	0.25	0.639	0.639	0.494
2	A2	0.494	0.433	0.25	1.07	0.881	1.7
3	A3	1.07	0.494	0.25	2.66	1.7	1.07
4	A4	0.333	0.333	0.25	0.799	1.48	2.23
5	A5	0.32	0.639	0.32	2.68	1.07	1.07
6	A6	0.881	1.07	0.32	0.881	1.7	0.799
7	A7	0.799	0.639	0.25	1.48	0.799	0.494
8	A8	11.9	11.9	14.6	11.9	11.9	11.9
9	A9	11.9	11.9	8.4	8.4	8.4	8.4
10	A10	11.9	11.9	11.9	8.4	11.9	8.4
11	A11	11.9	11.9	11.9	11.9	8.4	11.9
12	A12	11.9	11.9	11.9	8.4	11.9	11.9
13	A13	5.58	5.58	5.58	8.4	5.58	5.58
14	A14	5.58	5.58	5.58	5.58	5.58	5.58
15	A15	11.9	11.9	11.9	14.6	11.9	8.4
16	Height (in)	Invariable	Invariable	Invariable	131.03	131.62	113.90
Weight (lb.)		50295.90	50247.66	48985.05	42957.98	41589.25	39778.21
Max. Strength Ratio		0.8724	0.8689	0.8751	0.9865	0.9604	0.9516
No. of Analyses		20,000	20,000	19,800	10,000	10,000	8,900

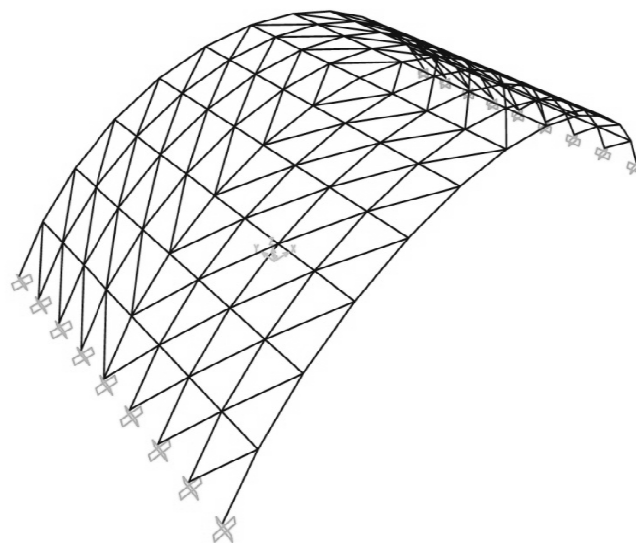
Table 4 Comparison of the best weights for the 173-bar single layer barrel vault frame

Optimization Problem	Best Weight (lb.)		
	CSS	MCSS	IMCSS
Size Optimization [16]	50295.90	50247.66	48985.05
Simultaneous Shape and Size Optimization	42957.98	41589.25	39778.21
Percent of reduction in best weights	14.59%	17.23%	18.80%

5.2. A 292-bar single layer barrel vault

This spatial structure which is shown in Fig. 9 has a three-way pattern [16]. The structure consists of 117 joints and 292 members. The problem has 31 design variables consists of size and shape variables. In the problem of size optimization, considering the symmetry of the geometry and loading conditions, all members are grouped into 30

independent size variables groups as shown in Fig. 9(b). For the problem of shape optimization, the lower and upper bounds of height as the only shape variable are 1.8 m and 18 m, respectively. The nodes are subjected to the displacement limits of ± 1.31 in (33 mm) in x, y directions and ± 1.97 in (50 mm) in z directions.



(a)

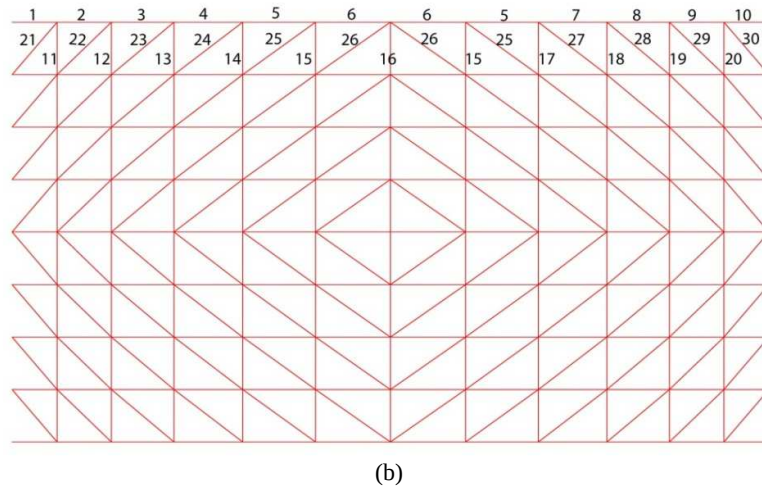


Fig. 9 The 292-bar single layer barrel vault frame: (a) 3-dimensional view, (b) Member groups in top view [16]

The configuration of this structure is as follows

- Span (S) = 36 m (1417.3 in)
- Height (H) = 8 m (393.7 in)
- Length (L) = 20 m (787.4 in)

According to the loading considerations in Section 4, three loading conditions are applied to this barrel vault as follows:

A uniform dead load of 100 kg/m^2 is applied on the roof. The applied snow load and wind load acting on this barrel vaults is respectively shown in Fig. 10 (a) and (b).

Table 5 is provided for comparison the results of the CSS, MCSS and IMCSS algorithms for this structure. The convergence history of all algorithms are shown in Fig. 11.

As shown in Table 5 the best weight of IMCSS algorithm is 51856.76 lb (23521.83 kg), while it is 57119.63 lb and 52773.58 lb for the CSS and MCSS algorithms. Although the CSS and MCSS algorithms find their best solutions in 13,200 and 12,500 analyses, the IMCSS algorithm obtains better solutions in 122 iterations (12,200 analyses).

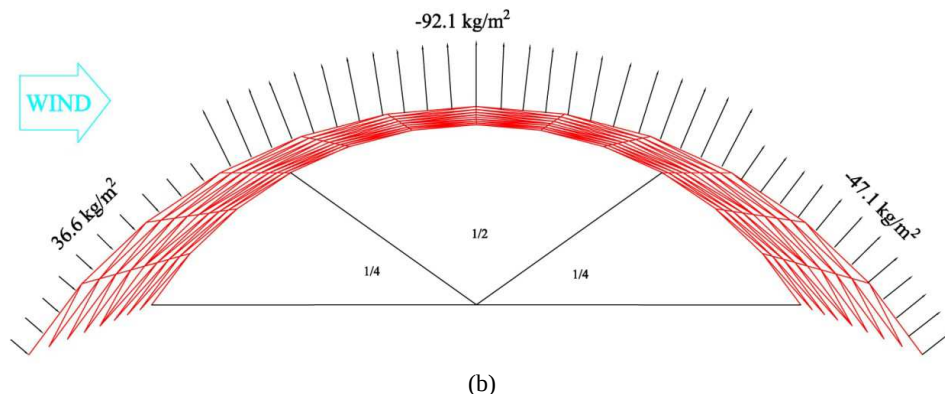
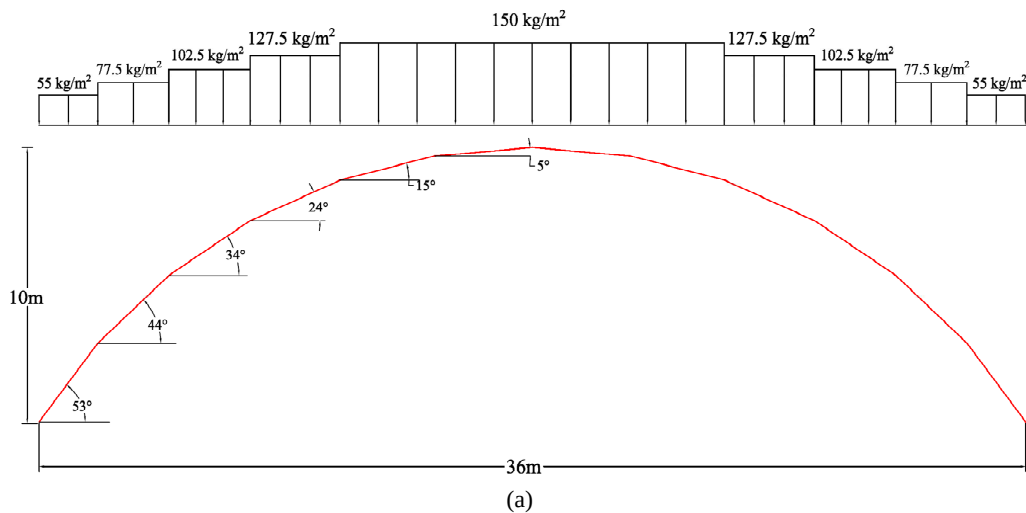
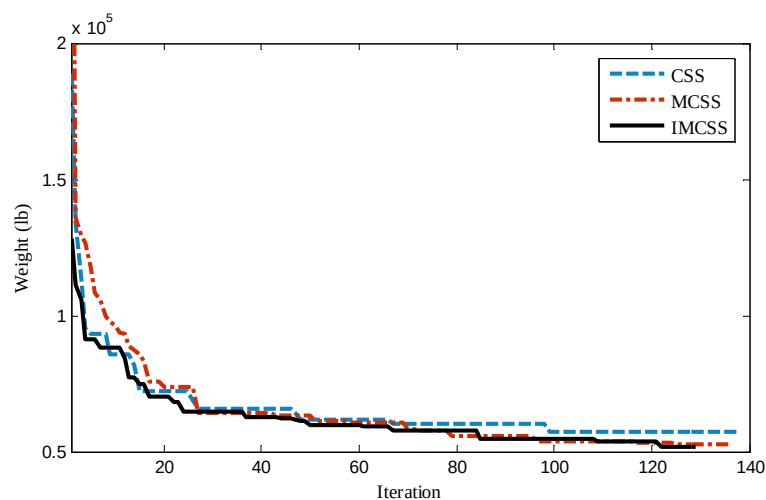


Fig. 10 The 292-bar single layer barrel vault frame subjected to: (a) Snow and (b) Wind loading [16]

Table 5 Optimal solutions for simultaneous shape and size optimization of the 292-bar barrel vault (in²)

Design Variables		CSS		MCSS		IMCSS	
		Section Name	Area (in. ²)	Section Name	Area (in. ²)	Section Name	Area (in. ²)
1	A1	'P12'	14.6	'P10'	11.9	P10	11.9
2	A2	'XP6'	8.4	'XP6'	8.4	P10	11.9
3	A3	'XP10'	16.1	'XP8'	12.8	XXP5	11.3
4	A4	'XXP5'	11.3	'P10'	11.9	XP6	8.4
5	A5	'XP6'	8.4	'XP5'	6.11	XP6	8.4
6	A6	'XP6'	8.4	'XP6'	8.4	XP6	8.4
7	A7	'XP6'	8.4	'P10'	11.9	P10	11.9
8	A8	'XXP5'	11.3	'XP6'	8.4	P10	11.9
9	A9	'XP6'	8.4	'XXP5'	11.3	P10	11.9
10	A10	'XP12'	19.2	'P12'	14.6	P12	14.6
11	A11	'XP2.5'	2.25	'P1.25'	0.669	XP3	3.02
12	A12	'XP3.5'	3.68	'P2.5'	1.7	P1	0.494
13	A13	'P2.5'	1.7	'XXP3'	5.47	XP1.5	1.07
14	A14	'P2.5'	1.7	'P1.25'	0.669	P1	0.494
15	A15	'XP2.5'	2.25	'XP2.5'	2.25	XP2.5	2.25
16	A16	'P2.5'	1.7	'P2.5'	1.7	XP3.5	3.68
17	A17	'P2.5'	1.7	'XP5'	6.11	P2.5	1.7
18	A18	'XP1.25'	0.881	'P6'	5.58	P1.5	0.799
19	A19	'XP3.5'	3.68	'P2.5'	1.7	P2.5	1.7
20	A20	'P0.75'	0.333	'XP0.5'	0.32	XP3	3.02
21	A21	'XP3'	3.02	'P3'	2.23	XP2	1.48
22	A22	'P4'	3.17	'XP4'	4.41	XP1.5	1.07
23	A23	'P2.5'	1.7	'P2.5'	1.7	XP1.5	1.07
24	A24	'P3'	2.23	'P3'	2.23	XP3	3.02
25	A25	'P2.5'	1.7	'XP2'	1.48	P3	2.23
26	A26	'P3'	2.23	'XP2'	1.48	P3	2.23
27	A27	'XP2.5'	2.25	'XP4'	4.41	XP3.5	3.68
28	A28	'P2.5'	1.7	'XP3'	3.02	P2.5	1.7
29	A29	'XP6'	8.4	'XP2'	1.48	P1.25	0.669
30	A30	'XP2.5'	2.25	'XP2.5'	2.25	XP1.25	0.881
31	Height	204.8791 in (5.20 m)		163.0436 in (4.14 m)		173.0666 in (4.40 m)	
Weight. lb.		57119.63		52773.58		51856.76	
Weight. Kg.		25909.03		23937.69		23521.83	
Max. Displacement (in)		1.5802		1.5008		1.4424	
Max. Strength Ratio		0.9413		0.9303		0.9746	
No. of Analyses		13,200		12,500		12,200	

**Fig. 11** Convergence history for the 292-bar single layer barrel vault frame using CSS, MCSS and IMCSS algorithms

The best value for height of this barrel vault from CSS, MCSS and IMCSS algorithms is 204.88 in, 163.04 in and

173.07 in, respectively. The best height-to-span ratios, therefore, obtained from CSS, MCSS and IMCSS

algorithms are 0.15, 0.12 and 0.12 respectively, which are approximately close to value of 0.17 from Parke's study.

Table 5 also shows the maximum displacement and strength ratios for all algorithms. The values of maximum strength ratio for CSS, MCSS and IMCSS algorithms are 0.9413, 0.9303 and 0.9746 respectively, and the values of maximum displacement are 1.5802 in, 1.5008 in and 1.4424 in, respectively. The strength ratios for all elements of the 292-bar single layer barrel vault algorithms are depicted in Fig. 12(a) through (c), and the maximum strength ratios for element groups of this structure are presented in Fig. 13(a) through (c) for optimal results of

CSS, MCSS and IMCSS algorithms, respectively.

As shown in Fig. 12 (a) to (c), all of strength ratios of elements are lower than 1, therefore, all of presented algorithms have no violation of constraints in their best solutions and the constraints are satisfied.

Table 6 draws a comparison between the results of present work on simultaneous shape and size optimization and those of previous study on size optimization [16] for this structure. On comparison of the best weights for presented algorithms shown in Table 7, the value of weight of structure has decreased by 16.4%, 17.23% and 17.65% via CSS, MCSS and IMCSS algorithms, respectively.

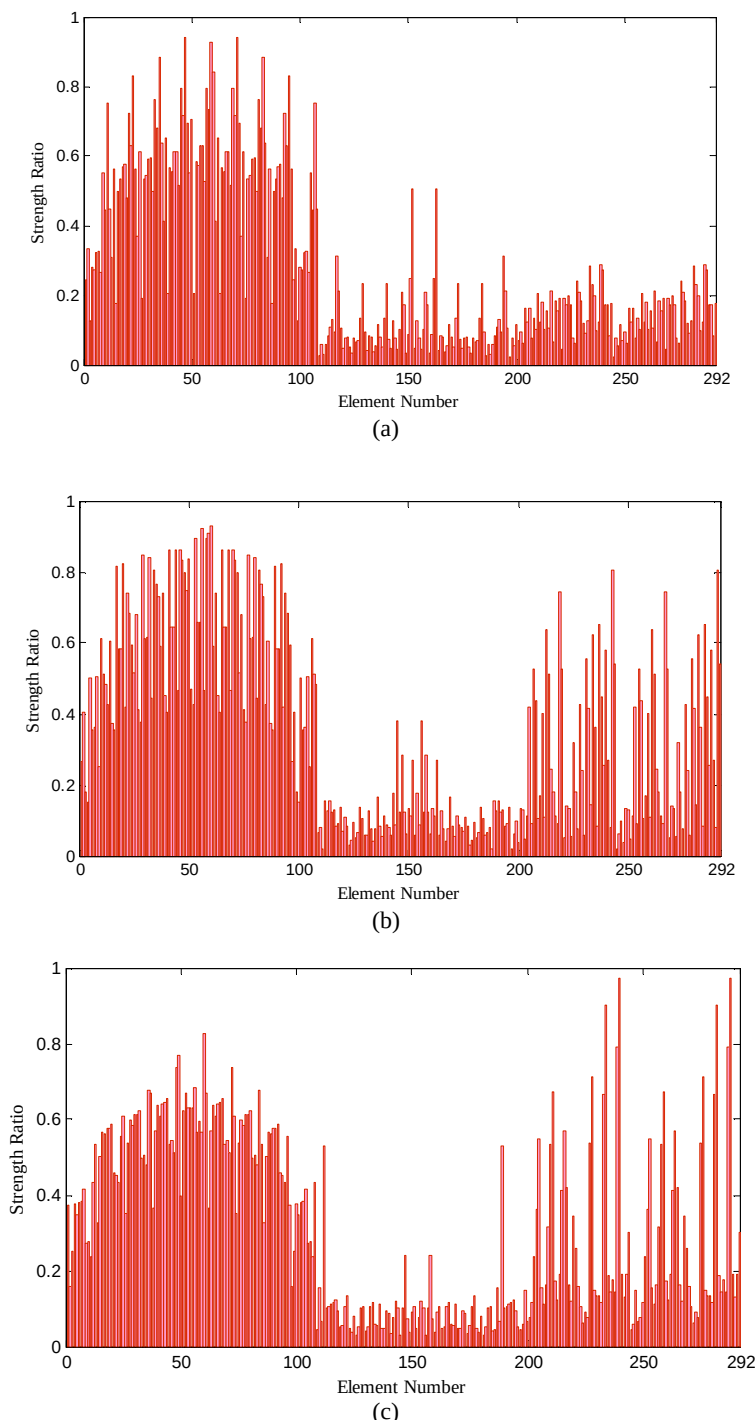


Fig. 12 Strength ratios for the elements of the 292-bar single layer barrel vault frame for optimal results of (a) CSS, (b) MCSS and (c) IMCSS algorithms

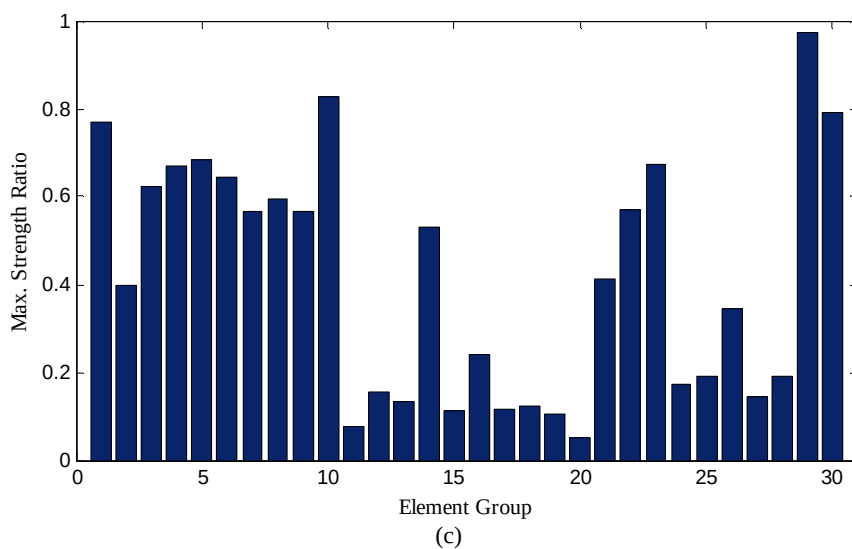
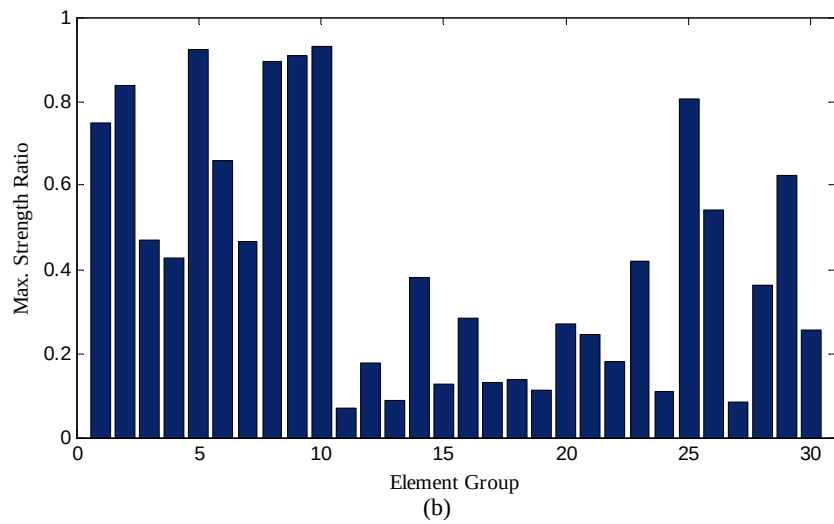
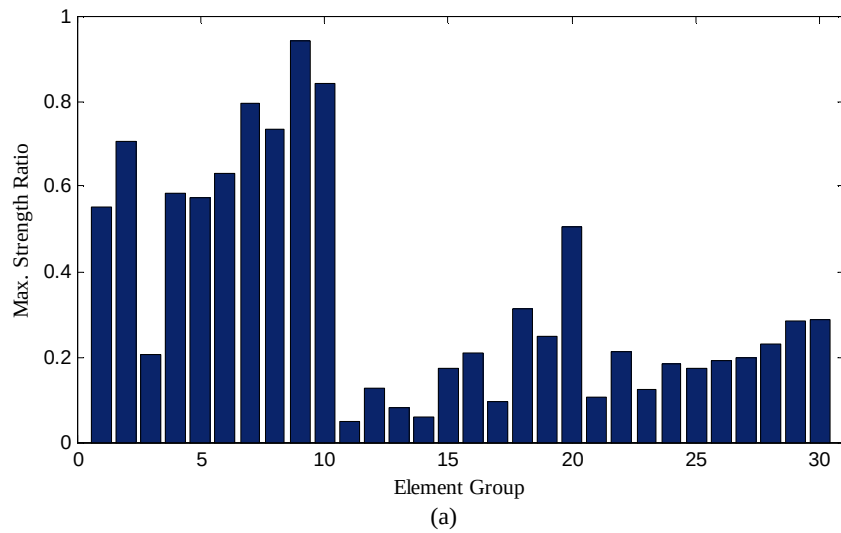


Fig. 13 Maximum strength ratios for element groups of the 292-bar single layer barrel vault frame for optimal results of (a) CSS, (b) MCSS and (c) IMCSS algorithms

Table 6 Comparison of the optimal solutions for the 292-bar single layer barrel vault frame

Design Variables		Kaveh <i>et al.</i> [16]			Present Work		
		Size Optimization			Simultaneous Shape and Size Optimization		
		CSS	MCSS	IMCSS	CSS	MCSS	IMCSS
1	A1	14.6	14.6	14.6	14.6	11.9	11.9
2	A2	11.9	8.4	8.4	8.4	8.4	11.9
3	A3	12.8	12.8	11.9	16.1	12.8	11.3
4	A4	5.58	14.6	8.4	11.3	11.9	8.4
5	A5	12.8	11.9	11.9	8.4	6.11	8.4
6	A6	11.9	11.9	11.9	8.4	8.4	8.4
7	A7	11.9	14.6	11.9	8.4	11.9	11.9
8	A8	14.6	16.1	14.6	11.3	8.4	11.9
9	A9	11.9	11.9	11.9	8.4	11.3	11.9
10	A10	19.2	19.2	14.6	19.2	14.6	14.6
11	A11	2.25	0.25	1.48	2.25	0.669	3.02
12	A12	0.669	0.433	0.799	3.68	1.7	0.494
13	A13	6.11	1.7	0.669	1.7	5.47	1.07
14	A14	3.68	0.639	0.799	1.7	0.669	0.494
15	A15	1.7	0.669	0.494	2.25	2.25	2.25
16	A16	3.17	1.07	0.799	1.7	1.7	3.68
17	A17	1.48	2.68	2.25	1.7	6.11	1.7
18	A18	1.48	1.07	0.669	0.881	5.58	0.799
19	A19	5.47	0.639	0.639	3.68	1.7	1.7
20	A20	4.3	2.23	1.48	0.333	0.32	3.02
21	A21	2.66	1.48	0.799	3.02	2.23	1.48
22	A22	2.25	1.07	1.07	3.17	4.41	1.07
23	A23	0.639	2.23	0.799	1.7	1.7	1.07
24	A24	1.48	1.7	1.07	2.23	2.23	3.02
25	A25	0.799	0.669	0.669	1.7	1.48	2.23
26	A26	1.07	0.669	0.881	2.23	1.48	2.23
27	A27	0.799	1.7	0.799	2.25	4.41	3.68
28	A28	1.48	2.23	0.799	1.7	3.02	1.7
29	A29	1.07	0.799	1.48	8.4	1.48	0.669
30	A30	2.68	0.799	12.8	2.25	2.25	0.881
31	Height (in)	Invariable	Invariable	Invariable	204.8 [^]	163.04	173.0 ^v
Weight (lb.)		68324.57	65892.33	62968.19	57119.63	52773.58	51856.76
Max. Strength Ratio		0.9527	0.8883	0.9939	0.9413	0.9303	0.9746
No. of Analyses		20,000	20,000	17,500	13,200	12,500	12,200

Table 7 Comparison of the best weights for the 292-bar single layer barrel vault frame

Optimization Problem	Best Weight (lb.)		
	CSS	MCSS	IMCSS
Size Optimization [16]	68324.57	65892.33	62968.19
Simultaneous Shape and Size Optimization	57119.63	52773.58	51856.76
Percent of reduction in best weights	16.40%	19.91%	17.65%

6. Concluding Remarks

This paper has applied an optimization approach which contains improved magnetic charged system search (IMCSS) and open application programming interface (OAPI) for simultaneous shape and size optimization of barrel vaults frames. In this approach, OAPI is utilized as a programming interface tool through programming language to manage the process of structural analysis during the optimization process and the IMCSS which is improved version of MCSS algorithm is used for achieving better solutions for the optimization problem.

In this study, two single layer barrel vault frames with different patterns are optimized via the presented

approach. In the process of optimization, contrary to size variables, shape is a continuous variable. In the case of shape optimization of this type of space structures, since all of the nodal coordinates as the shape variables are dependent on the height-to-span ratio of the barrel vault, height is considered as the only shape variable in a constant span of barrel vault.

On comparison, the best height-to-span ratios of barrel vaults under static loading conditions obtained from CSS, MCSS and IMCSS algorithms are approximately close to value of 0.17 from comparative study carried out by Parke. Furthermore, as seen in the results, different patterns of barrel vaults have different effects on the value of best height-to-span ratio. Moreover, in comparison to CSS and

MCSS algorithms, IMCSS has found more optimal values for the weight of structures in a lower number of analyses.

Since SAP2000 is a powerful software in modeling, analyzing and designing of large-scale spatial structures, OAPI would be a profit interface tool between this software and MATLAB in the process of structural optimization.

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